

Some Neutrino Astrophysics, Looking Forward

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Abstract. Theoretical and experimental constructions together make a compelling case that the discovery of high-energy ($\gtrsim 10^{15}$ eV) astrophysical neutrinos is imminent. In this talk, we investigate some of the reasons for excitement. We focus on the larger cosmic neutrino flux expected due to the recently-argued lower-energy cosmic-ray crossover from galactic to extra-galactic origin; and on various aspects of source dynamics that can be probed by neutrino flavor physics. We conclude with a brief overview of the information contained in the fluctuations and correlations expected in neutrino flavor data.

1. Introduction

There is redundancy in the title I have been assigned for this talk. *Any* physics in a neutrino telescope would constitute new physics! To date, extra-terrestrial neutrinos have been detected from just two sources, each relatively nearby, the sun and supernova 1987a. The characteristic energies of these neutrinos are a few to tens of MeV. Clearly there is room in the Universe for further, more energetic, more exotic astrophysical neutrino discoveries.

Solar neutrinos are identifiable because their flux is enormous, of order $10^{12} \text{ cm}^{-2} \text{ s}^{-1}$, and because they scatter target electrons forward, which allows directional reconstruction.¹ The SN87a neutrinos were identified even though their transit distance was 50 kpc, because their flux was tremendous and because the time of arrival was known from the accompanying photon signals. The goal of neutrino astronomy is to identify further neutrino sources, both galactic and extra-galactic, and eventually to measure a spectrum of diffuse neutrinos. The goal appears to be within reach, starting now and extending through the decade. DUMAND in the ocean off of Hawaii was the prototype neutrino telescope, followed by BAIKAL in deep fresh water, AMANDA and RICE at the South Pole, ANITA above the South Pole, and IceCube and AURA under construction at the Pole. The Antarctic Ice Cap is becoming an Antarctic Neutrino Trap. ANTARES, NESTOR, and NEMO are under construction in the Mediterranean Sea. KM3net is envisioned for the Mediterranean, ARIANNA for the Antarctic's Ross ice shelf, EUSO (now JEM-EUSO) for the International Space Station (ISS), and SALSA for a giant salt dome, probably along the US Gulf Coast. As these bigger and better neutrino projects climb to completion, the anticipation of physics discovery climbs as well.

Generally, extra-terrestrial fluxes of neutrinos are expected to exceed the neutrino flux produced in our atmosphere (from the decay of charged pions, themselves copiously produced by cosmic ray interactions in air) when the neutrino energy exceeds about 100 TeV. Since

¹ Coincidentally, $10^{12} \text{ cm}^{-2} \text{ s}^{-1}$ is also the present day flux of relic neutrinos left over from the early-Universe thermal epoch produced by the Big Bang.

the Galactic cosmic-ray spectrum extends to about 10^{18} eV, and the extra-galactic cosmic-ray spectrum to a bit beyond 10^{20} eV, there is extreme optimism that an accompanying neutrino flux will be found up to comparable energies. Thus, the search for extra-terrestrial neutrinos spans the energies from ~ 100 TeV to $\sim 10^{20}$ eV. The latter energy represents a center-of-momentum (CM) energy of almost a PeV, three orders of magnitude beyond the lepton-nucleon CM available at the HERA machine in DESY, Germany.²

The weak-ness of the neutrino cross-section translates into huge-ness for neutrino detector volumes. For detection of PeV (10^{15} eV) neutrinos, as might come from AGN sources, a minimum of a gigaton of mass is probably required; in ice or water or salt, this translates into a km^3 of volume. For detection of neutrinos at 10^{19} eV, as might come from the GZK process, from Gamma-Ray Bursts and other astrophysical sources, a teraton of mass is probably needed. For teraton detection, space-based observation of showers in our atmosphere as proposed by EUSO seems to be the best possibility, although teraton studies of a neutrino events via the less direct approach of radio detection is possible. Radio telescope have monitored the moon and will continue to do so, and balloon-borne radio antennae are monitoring the polar ice cap. Eventually, radio antennae in salt domes may attain teraton sensitivity. Of course, there may be truly exotic physics providing surprising fluxes awaiting discovery; we cannot know this until we look.

The weak-ness of the neutrino cross-section is certainly a disadvantage for experimental design. However, this same weak-ness is at the heart of the tremendous promise of neutrino astronomy. The advantages manifest themselves in four ways:

- (i) Neutrinos are not absorbed on the diffuse radiation backgrounds (DRBs) that permeate our Universe. In contrast, photons are absorbed above the threshold energy for $\gamma + \gamma(\text{DRB}) \rightarrow e^+e^-$, and cosmic-rays are absorbed at the famous Greisen-Zatsepin-Kuzmin (GZK) threshold $\sim 5 \times 10^{19}$ eV for the process $N + \gamma(\text{CMB}) \rightarrow \pi + N'$.
- (ii) Consequently, neutrinos may arrive from very distant cosmic sources, characterized by red-shifts $z > 1$.
- (iii) Neutrinos may emerge from deep within the environment of a compact source, almost down to the Schwarzschild radius of an accreting black hole (BH). Of course, the neutrino energy will red-shift upon emergence from the potential well of a compact source. The depths probed by neutrinos may bring to us unique information. A relevant analogy is the emissions of our Sun. Solar photons come from the very outer layer of chromosphere, where they have a mean free path of about a centimeter; solar neutrinos come from the central fusion core.
- (iv) Neutrinos are not deflected by magnetic fields, and so will point back to their sources. It is this latter fact that enables one to go beyond neutrino astrophysics to contemplate neutrino astronomy.

Finally, I would like to add one more unique feature of neutrinos. This feature may turn out to be the most important, so I give it a capital letter.

(V) Neutrinos come in three flavors, ν_e , ν_μ , and ν_τ . The flavor ratios carry special information, as we will discuss below. One may think of the flavor information as the neutrino analog of the polarization information carried by photons (used e.g. to study cosmic magnetism via Faraday rotation), or the atomic number A carried by the cosmic-rays (used e.g. to differentiate proposed mechanisms of cosmic-ray origin.).

Below I will discuss the use of neutrino flavor to discriminate competing models for source dynamics. But first I would like to discuss some recent work on the role neutrinos may play in settling the “galactic/extra-galactic” cosmic-ray debate. The debate centers on the question,

² It is worth mentioning that cosmic rays offer energies up to 10^7 times higher than will be available at the most energetic person-made accelerator, the LHC. And the only cost involved is the price of the detector - Nature provides the beam for free!

at what energy does the extra-galactic flux of cosmic-rays overtake the galactic flux. A lower-energy crossover seems probable in many opinions [?], including my own. This brings good news for neutrino astrophysicists, for we infer a more powerful extragalactic neutrino flux as a result.

2. Neutrinos and the Galactic/Extra-Galactic Crossover Debate

There are three statistically significant breaks in the power-law spectrum of cosmic-rays [?]. The first, called the “knee”, is a steepening near a PeV. It is thought that this break signifies the maximum energy to which protons are accelerated in the shocks associated with supernova remnants. We do not concern ourselves with this spectral break. Next comes the “second knee”, a further steepening of the cosmic-ray spectrum just below 10^{18} eV. And the final break is the “ankle”, a hardening of the spectrum beginning just below 10^{19} eV. Until recently, the feeling was that the second knee was not very significant theoretically, and the ankle exhibited the energy at which the extra-galactic flux overtook the galactic flux. In contrast, the current belief among many is that the second knee is very significant theoretically, and the ankle less so. V. Berezhinsky and his collaborators [?] have pointed out that the purely electromagnetic energy-loss process $p + \gamma(\text{CMB}) \rightarrow p + e^+e^-$ naturally provides a dip whose origin in energy coincides with the second knee, and a minimum at the energy coinciding with the ankle. An inference to be drawn from this notion is that the power in the extra-galactic flux is much larger than previously thought; with the cosmic-ray flux falling roughly as E^{-3} in this region, a cross-over energy lower by an order of magnitude requires an increased normalization of the extra-galactic flux (power) by three (two) orders of magnitude. This in turn implies a larger pion emission at the source, and therefore a larger emitted neutrino flux. A reasonable guess [?] is that at the source, similar power is emitted in nucleons and in pions. Since the charged pions decay to neutrinos, we now expect the neutrino fluxes produced at the cosmic sources to be larger by a factor ~ 100 . This increase in neutrino power is large enough to dominate over the calculated GZK neutrino flux. Thereby, prospects for extreme-energy (EE) neutrino detection in planned detectors is brighter.

In Fig. (??) are shown fits [?] to the EE cosmic-ray spectrum. The injection spectrum at the source is parameterized as $E_\nu^{-\gamma} (1+z)^n$, with γ being the common spectral index of the sources, and n characterizing source-evolution at higher red-shifts z . Visible is the spectacular accommodation of the dip about the ankle by the $p + \gamma(\text{CMB}) \rightarrow p + e^+e^-$ process.

In Fig. (??) is shown the prediction for the neutrino flux today, produced directly by these high- z cosmic sources. The spectral index of the neutrinos, ~ 2.5 is the same as the fitted index for the cosmic-rays. For comparison, we show too the original E^{-2} benchmark flux of Waxman and Bahcall (WB) [?]. Both the WB flux and the new early-crossover flux are normalized to the cosmic-ray flux by the assumption of equal power in nucleon and pion emission. One can see that the new estimate of neutrino flux, coming from the low-crossover hypothesis, is considerably larger than the older WB estimate, coming from the higher-crossover assumption.³ One can also see that the neutrino flux coming directly from the sources equals or exceeds the GZK flux at all energies. And finally, one can see that the sensitivity of the AMANDA-B10 telescope, extrapolated from PeV to EeV energies, almost reaches the predicted flux. Since the IceCube telescope, now under construction, will have a sensitivity larger than that of AMANDA-B10 by a factor between one and two orders of magnitude, IceCube will be able to validate or invalidate the low-crossover hypothesis if the spectrum extrapolation is valid (but see the previous footnote). The hope, of course, is that the higher neutrino flux associated with the lower-crossover will be

³ The larger index for the neutrino spectrum means that more power is required at the sources. In order to not violate power limits, the neutrino spectrum must cutoff not too far below the second knee. An example of a natural cutoff is provided in photo-pion production $N + \gamma \rightarrow N + \pi$ of neutrinos, where the energy threshold 145 MeV for the photon in the nucleon rest frame translates into a nucleon threshold $\sim 10^{17}$ (eV/ $\langle\omega\rangle$) eV in the lab frame, where $\langle\omega\rangle$ is the mean photon energy in the lab frame.

validated by “lighting up” various neutrino telescopes.

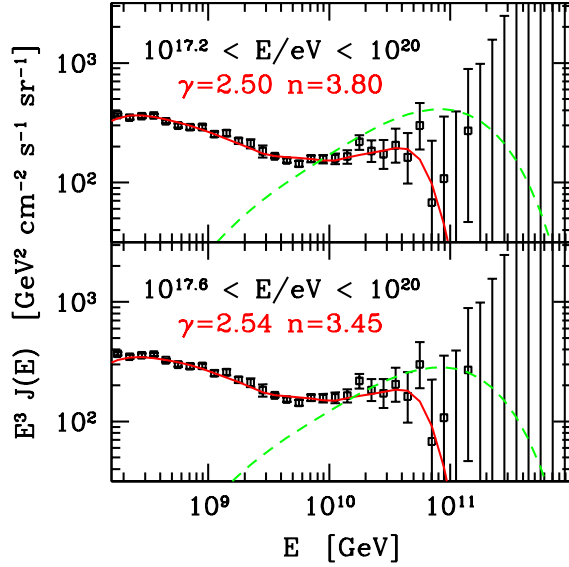


Figure 1. Best fit to HiRes data (solid), assuming dominance of the extra-galactic component above $E_- = 10^{8.2}$ GeV (top) and $E_- = 10^{8.6}$ GeV (bottom). Also shown is the associated cosmogenic neutrino flux for all flavors (dashed).

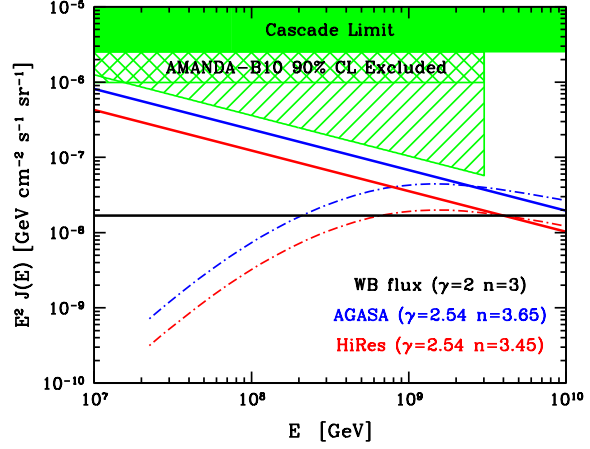


Figure 2. Falling solid lines indicate the expected flavor-summed neutrino flux for low crossover energy ($10^{8.6}$ GeV), normalized to HiRes (lower) and AGASA (upper) data; horizontal line indicates the WB prediction for high crossover energy ($\sim 10^{10}$ GeV). Dash-dot lines indicate the predicted cosmogenic neutrino fluxes accompanying the falling solid lines. The single hatched region is the 90%CL AMANDA exclusion (AMANDA [?]) extrapolated with $E^{-2.54}$ for the low crossover model; the cross-hatched region is the exclusion for an E^{-2} extrapolation.

3. Neutrino Flavor Propagation and Wave Packet Decoherence

Individual cosmic ray neutrinos from cosmically-distant sources will decohere in transit to Earth, as their spatial wave functions separate into non-overlapping mass eigenstates. Consequently, the flavor probabilities to be measured at Earth are given by classical considerations. Some neutrino telescopes will have the capability to partition incident neutrinos into flavors, based on the differing topologies observed for interacting ν_e 's, ν_μ 's, and ν_τ 's [?]. In this section we focus on the inference of the initial flavor ratios at the source from the flavor ratios measured at Earth. In the next section we discuss the implications of flavor discrimination for source dynamics.

Assume that the cosmic neutrino beam is produced with initial flavor weights w_α , normalized to $\sum w_\alpha = 1$. Then the initial neutrino beam is described by the density matrix $\rho(0) = \sum_\alpha w_\alpha |\alpha\rangle \langle\alpha|$. Greek indices label states in the flavor basis, and roman indices will label states in the mass basis. The flavor state $|\alpha\rangle$ is related to the mass-eigenstates by the mixing matrix U : $|\alpha\rangle = \sum_j U_{\alpha j} |j\rangle$, or, $U_{\alpha j} = \langle j|\alpha\rangle = \langle\alpha|j\rangle^*$. Thus, the initial neutrino density in the mass basis is $\rho(0) = \sum_{\alpha,j,k} w_\alpha U_{\alpha j} U_{\alpha k}^* |j\rangle \langle k|$. In traveling a cosmic distance D , the off-diagonal elements of the density matrix acquire a phase $e^{-i(E_j - E_k)D/\hbar c} = e^{-i\delta m_{jk}^2 D/2E}$, $\delta m_{jk}^2 \equiv m_j^2 - m_k^2$. However, over a cosmic distance, the beam decoheres into separated mass states; i.e., the time-of-flights of the three mass states constituting the single neutrino are sufficiently different that the single neutrino wave packet spreads into three non-overlapping components. Consequently, the quantum mechanical interferences vanish and probabilities become classical. In our formalism,

this means that the off-diagonal elements of the neutrino density matrix in the mass basis vanish. We are left with the decohered density matrix

$$\rho(D) = \sum_{\alpha,j} w_{\alpha} |U_{\alpha j}|^2 |j\rangle \langle j| = \sum_j w_j |j\rangle \langle j|. \quad (1)$$

Here, we have defined the weight for the mass-basis as $w_j \equiv \sum_{\alpha} w_{\alpha} |U_{\alpha j}|^2$, again normalized to $\sum_j w_j = 1$.

After decoherence is complete, no interference remains, and T and CP are consequently, effectively good symmetries. This implies that flavor-changing probabilities are the same for the neutrino and the anti-neutrino channels, and for channels related by time-reversal.

When a neutrino is detected in a charged-current process at Earth, the probability that flavor β will be observed is

$$p_{\beta} = \langle \beta | \rho(D) | \beta \rangle = \sum_{\alpha,j} w_{\alpha} |U_{\alpha j}|^2 |U_{\beta j}|^2 = \sum_j w_j |U_{\beta j}|^2. \quad (2)$$

Since the $|U_{\beta j}|^2$ are known from terrestrial oscillation studies, the new information from neutrino astrophysics is contained in the set $\{w_j\}$, or $\{w_{\alpha}\}$.

The role of *experimental* neutrino astrophysics is to infer the $\{w_j\}$. An *ideal experiment* on Earth that measures N neutrino events will correctly partition these events into n_e, n_{μ}, n_{τ} flavor bins, with $\sum_{\alpha} n_{\alpha} = N$. From these three idealized bins, one can infer the values of w_1, w_2, w_3 . However, to date no high-energy ($\gtrsim 100$ TeV) extragalactic neutrino events have yet been observed in prototype experiments, and so it is becoming clear that the number of events N to be gathered in near-future experiments will be small. Consequently, the Poisson fluctuations in the numbers $\{n_{\alpha}\}$ will be relatively large. Given the paucity of information expected in the early era of neutrino astrophysics, it will be wise to use all the information that Nature may give to us to improve the inference of $\{w_j\}$. This will include the information present in the expected fluctuations and cross-fluctuations of the counts $\{n_{\alpha}\}$ [?].

First let us outline what can be naively inferred without paying attention to fluctuations. The $|\langle \alpha | j \rangle|^2 = |U_{\alpha j}|^2$ are just the classical probabilities for projecting neutrino ν_{α} into the j^{th} mass state. It is convenient to collect these classical probabilities into a matrix P , whose elements are $P_{\alpha j} \equiv |U_{\alpha j}|^2$. For three neutrinos, the P matrix is explicitly

$$P = \begin{pmatrix} |U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\ |U_{\mu 1}|^2 & |U_{\mu 2}|^2 & |U_{\mu 3}|^2 \\ |U_{\tau 1}|^2 & |U_{\tau 2}|^2 & |U_{\tau 3}|^2 \end{pmatrix}. \quad (3)$$

It is clear that unitarity of the U matrix implies that elements in a row or column of the P matrix sum to unity. There is a linear relation between the “unit” vectors $\vec{w}_j = (w_1, w_2, w_3)$ or $\vec{w}_{\alpha} = (w_e, w_{\mu}, w_{\tau})$ and mean $\vec{n} = N^{-1} (\langle n_e \rangle, \langle n_{\mu} \rangle, \langle n_{\tau} \rangle) = P \vec{w}_j = \mathcal{P} \vec{w}_{\alpha}$, with $\mathcal{P} \equiv P P^{\text{T}}$ called the “flavor propagation matrix”. If P is an invertible matrix, then we also have $\vec{w}_j = P^{-1} \vec{n}$ or $\vec{w}_{\alpha} = \mathcal{P}^{-1} \vec{n}$.

The “tri-bimaximal” model with $\theta_{32} = 45^{\circ}$ (maximal mixing), $\tan^2 \theta_{12} = 1/2$, and $\theta_{13} \approx 0$, describes known neutrino phenomenology. It leads to

$$P \approx \frac{1}{6} \begin{pmatrix} 4 & 2 & 0 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \quad \text{and} \quad \mathcal{P} = P P^{\text{T}} = \frac{1}{18} \begin{pmatrix} 10 & 4 & 4 \\ 4 & 7 & 7 \\ 4 & 7 & 7 \end{pmatrix}. \quad (4)$$

Since the lower two rows of P are proportional, in fact identical, the determinant of P vanishes

and the matrix is not invertible.⁴ This may be traced back to the ν_μ - ν_τ symmetry implicit in the tri-bimaximal model.

We call attention to the fact that, regardless of the initial flavor composition of the cosmic beam, a relatively large fraction of each of the ν_e , ν_μ , and ν_τ will be present in the beam at Earth. Such is the inevitable result of bi-large mixing, which leads to the quasi-uniform distribution of values for all matrix elements in \mathcal{P} . This is good for neutrino telescopes, because different telescopes have different sensitivities and systematics for the various neutrino flavors.

4. Relevance of Neutrino Flavor Measurements to Astrophysics

There are several examples of astrophysics and neutrino physics which can be gleaned from a measurement of the neutrino flavor ratios at Earth. These include

(i) Search for a galactic β -beam [?]. If heavy nuclei are accelerated, then photo-disintegration at the source is expected to liberate some neutrons. Neutrons are not deflected by magnetic fields, and so they should point back to their sources. The mean path for neutrons is about 10 kpc ($E_n/10^{18}$ eV). Therefore, neutrons with energies near an EeV will arrive at Earth, while those of lower energy will β -decay intransit to produce a pure $\bar{\nu}_e$ beam, which will also point back to the source. Processing the β -beam vector $w_\alpha = (1, 0, 0)$ through the \mathcal{P} matrix gives the expected (anti)neutrino flavor ratios at Earth as (5,2,2). *This is the flavor signature for a cosmic β -beam.*

(ii) Search for pionic sources of neutrinos. The complete decay chain for a charged pion $\pi^\pm \rightarrow \mu + \nu_\mu \rightarrow e + \nu_e + 2\nu_\mu$ produces an initial flavor vector (unnormalized) $w_\alpha = (1, 2, 0)$. Processing this through \mathcal{P} leads to Earthly flavors of (1,1,1), i.e. complete flavor democracy. *This is the flavor signature for cosmic neutrino production via pion modes.* In fact, there is a “democracy theorem” which says that if the mixing angle θ_{32} is maximal, and $\Re(U_{e3}) = 0$, then regardless of θ_{12} , any initial combination of ν_μ and ν_τ will decohere to equal ν_μ and ν_τ . The known ranges of θ_{32} and U_{e3} satisfy the requirements of this theorem to a very good approximation. A corollary to this theorem is that if, in addition, the initial flavor ratio of ν_e is 1/3, then flavor democracy is the final result after decohering. Pion decay satisfies this latter condition.

(iii) The μ^\pm lifetime is about 100 times longer than the π^\pm lifetime. In some cosmic environments, the combination of magnetic field and time dilation $\sim E_\mu/m_\mu$ will cause the muon to synchrotron radiate a significant fraction of its energy before it decays. The result is effectively a termination of the pion decay chain to just $\pi^\pm \rightarrow \mu^\pm + \nu_\mu$. In this case, the initial flavor vector is (0,1,0), and the corresponding flavor vector at Earth is (4,7,7). Observation of a transition with increasing energy, from (1,1,1) to (4,7,7) would indicate the strength of the magnetic field at the cosmic source [?].

(iv) The pions at the source may be produced by pp or $p\gamma$ collisions. In the pp case, π^+ and π^- are made in nearly equal numbers over a large rapidity plateau, the democracy theorem is satisfied, and the expected flavors at Earth are (1,1,1) for both neutrino and antineutrino; in particular, the $\bar{\nu}_e$ fraction at Earth is 1/6. In the $p\gamma$ case, mainly π^+ 's are made via the excitation and decay of the Δ^+ resonance, and there results a neutrino-antineutrino asymmetry: the initial flavor content is (1,1,0) for the two neutrinos per π^+ , but (0,1,0) for the initial *antineutrino* per π^+ . The flavor contents expected at Earth are (14,11,11)/18 and (4,7,7)/18, respectively, which implies a $\bar{\nu}_e$ ratio of just 2/27, less than half that of the pp case. Furthermore, models for the two modes suggest that 2 to 3 times more power is emitted in pions in the pp case than in the $p\gamma$ case. The net result is that the $\bar{\nu}_e$ signal should be observable at IceCube via the resonant process $\bar{\nu}_e(6.4\text{PeV}) + e^- \rightarrow W^-$ for the pp origin, but not for the $p\gamma$ origin [?].

(v) One possible point of view regarding quantum gravity is that spacetime fluctuations behave

⁴ In general, four elements of P are independent, with no more than two chosen from any single row or column; labeling these A, B, C, D , the determinant (up to a sign) is $\frac{1}{3}(B+C-A-D) + (AD-BC)$, where A and D (B and C) share neither a row nor a column.

as virtual mini-BHs. Since BHs donot conserve global quantum numbers, a traveling particle interacting randomly with these virtual BHs would lose all memory of global quantum numbers. For the neutrino, this means that flavor would be randomized, replacing any initial flavor asymmetry with the democratic value (1,1,1). Thus, any measurement of a flavor ratio other than (1,1,1) would put a strong limit on this brand of quantum gravity [?].

Related topics of high interest to me come from the role that flavor may play in revealing exotic neutrino properties. These include astrophysical searches for a finite neutrino lifetime [?], and for a pseudo-Dirac mass splitting between left-handed (“active”) neutrino states and right-handed (“sterile”) states [?]. I do not have the space here to develop these topics.

5. Statistics with Small Event Number – the Trinomial Flavor Distribution

We alluded to the fact that important information may be carried not only by mean numbers of events types, but also by expected statistical fluctuations and correlations. This final section explores this concept.

The probability for partitioning N neutrino events among the three flavors is given by the trinomial distribution

$$P(N; n_e, n_\mu, n_\tau) = N! \frac{(p_e)^{n_e}}{n_e!} \frac{(p_\mu)^{n_\mu}}{n_\mu!} \frac{(p_\tau)^{n_\tau}}{n_\tau!}, \quad (5)$$

where the p_β are the expected values given by Eq. 2. From this general expression, one readily calculates the mean values $\langle n_\beta \rangle = p_\beta N$, from which one infers that mean \vec{n} defined earlier is equal to $\vec{p} = (p_e, p_\mu, p_\tau)$. One also readily calculates the variance to be

$$\langle n_\beta^2 \rangle - \langle n_\beta \rangle^2 = p_\beta (1 - p_\beta) N, \quad (6)$$

the cross correlations to be

$$\langle n_\alpha n_\beta \rangle - \langle n_\alpha \rangle \langle n_\beta \rangle = -p_\alpha p_\beta N, \quad \text{for } \alpha \neq \beta, \quad (7)$$

and the triple correlation to be

$$\langle n_e n_\mu n_\tau \rangle - \langle n_e \rangle \langle n_\mu \rangle \langle n_\tau \rangle = -p_e p_\mu p_\tau N (3N - 2). \quad (8)$$

Clearly, the variances and cross-correlations contain further information on the values of the expected probabilities p_β .

The statistical fluctuations are given as

$$n_\beta = \langle n_\beta \rangle + \delta_\beta, \quad \text{with } \delta_e + \delta_\mu + \delta_\tau = 0. \quad (9)$$

In terms of δ_β , we may recast Eqs. 6, 7, and 8 as

$$\langle \delta_\beta^2 \rangle = p_\beta (1 - p_\beta) N, \quad \langle \delta_\alpha \delta_\beta \rangle = -p_\alpha p_\beta N, \quad \langle \delta_e \delta_\mu \delta_\tau \rangle = 2 p_e p_\mu p_\tau N. \quad (10)$$

From the signs in Eq. 10, it is clear that of the expected fluctuations, one is positive (upwards) and two are negative (downwards). From the zero sum for the δ 's, it is then also clear that the largest fluctuation is expected to be the positive one.

Given a data set, one may use the moment formulas here to obtain a best-fit inference of the $\{w_j\}$ or $\{w_\alpha\}$. However, it has been known since the early days of bubble chamber physics that a maximum likelihood fit of theoretical expectations to data maximizes the information extracted from the data [?]. Accordingly, the moment distributions outlined here may be viewed as merely illustrative of the value inherent in higher order statistics.

6. Summary Statement

The ANITA experiment has just launched its balloon-borne radio detectors on December 15, 2006; in 45 days the flight will terminate and data analysis will begin. IceCube continues to implant its optical modules in the deep polar ice. The Japanese scientific community has recently embraced EUSO as a desirable mission for its module on the ISS. The discovery of astrophysical neutrinos seems imminent. Related theoretical considerations, some discussed in this report, will soon transcend speculation and join a phenomenological endeavor of great promise.

- [1] A recent review of the cosmic-ray spectrum, including a good discussion of the crossover, comes from the master himself, A. M. Hillas, arXiv:astro-ph/0607109.
- [2] V. Berezhinsky, A. Z. Gazizov and S. I. Grigorieva, arXiv:hep-ph/0204357.
- [3] E. Waxman and J. N. Bahcall, Phys. Rev. D **59**, 023002 (1999). [arXiv:hep-ph/9807282].
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- [5] M. Ackermann *et al.*, Astropart. Phys. **22**, 339 (2005).
- [6] See, e.g., talks at this conference by T. DeYoung and by D. Cowen.
- [7] The importance of mining fluctuations for information in the face of limited statistics was brought to my attention by S. Weinberg, during a seminar I gave recently at UT-Austin.
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- [12] J. F. Beacom *et al.*, Phys. Rev. Lett. **90**, 181301 (2003) [arXiv:hep-ph/0211305].
- [13] J. F. Beacom *et al.*, Phys. Rev. Lett. **92**, 011101 (2004) [arXiv:hep-ph/0307151].
- [14] I thank Med Webster for enlightening me on the topic of data fitting with sparse statistics.