

Spacetime foam at a TeV

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Abstract. Motivated by recent interest in TeV-scale gravity and especially by the possibility of fast baryon decay mediated by virtual black holes, we study another dangerous aspect of spacetime foam interactions: lepton flavor violation. We correlate existing limits on gravity-induced decoherence in the neutrino sector with a lower bound on the scale of quantum gravity, and find that if spacetime foam interactions do not allow an S -matrix description the UV cutoff is well beyond the electroweak scale. This suggests that string theory provides the appropriate framework for description of quantum gravity at the TeV-scale.

What is the meaning of quantum gravity? It means that spacetime itself is subject to quantum laws, necessitating inherent fluctuations in the fabric (metric and topology) of space and time. These microscopic boiling bubbles force on spacetime a foam-like structure [1]. A heuristic example pictures spacetime to be filled with tiny virtual black holes that pop in and out of existence on a timescale allowed by Heisenberg's uncertainty principle [2]. These black holes conserve energy, angular momentum, and electric and color charge (unbroken gauged quantum numbers), but they *are believed* not to conserve global quantum numbers. If this is the case, the transition between initial and final density matrices associated with black hole formation and evaporation is not factorizable into products of S -matrix elements and their hermitian conjugates. The evolution of such a quantum system is characterized by a superscattering operator \mathbb{S} that maps initial pure states to final mixed states, $\rho_{\text{out}} = \mathbb{S}\rho_{\text{in}}$, with $\mathbb{S} \neq S^\dagger S$ [3]. In other words, there *may be* a loss of quantum information across the black hole event horizons, providing an environment that can induce decoherence of apparently isolated matter systems.

In recent years much attention has been devoted to TeV-scale gravity models [4], as they provide an economic explanation of the hierarchy between the Planck and electroweak mass scales. In the canonical example, the Standard Model (SM) fields are confined to a four dimensional world (corresponding to our apparent universe), while gravity spills into large spatial compact dimensions without conflicting with experimental bounds [5]. Of particular interest is the question whether fast baryon decay can proceed via virtual black hole states in the spacetime foam [6]. The process is envisioned as the simultaneous absorption of two quarks into the black hole, followed without memory of the initial state by the thermal emission of an antiquark and a lepton, entailing a change in the global baryon and lepton quantum numbers $qq \rightarrow \bar{q}l$. The probability that two quarks in a proton of size $\Lambda_{\text{QCD}}^{-1}$ pass within a fundamental Planck length, within the Heisenberg lifetime uncertainty of the black hole is $\propto (\Lambda_{\text{QCD}}/M_{\text{QG}})^4$, where M_{QG} is the gravitational UV cutoff. Thus, the present limit on the proton lifetime, $\tau_p \sim 10^{33}$ yr [7], implies [6]

$$M_{\text{QG}} > 10^{16} \text{ GeV} . \tag{1}$$

Interactions through the higher dimensional QQQ operator can be prevented if one separates the quark and lepton fields far enough in an extra dimension so that their wave function overlap is exponentially suppressed [8]. Within this picture one may imagine that quark and leptons are fields localized on different branes at the boundaries of a thick wall, but of course gauge and higgs fields are free to propagate inside the wall so that quarks and leptons interact with one another through SM interactions. This division will certainly inhibit fast baryon decay. However, violation of global quantum numbers for processes with spacetime foam black holes on the lepton brane should proceed without wave function suppression. If there is loss of quantum information across event horizons, an interesting quantitative measurement of interaction with the spacetime foam emerges in its effect on neutrino oscillations: interaction with the virtual black holes introduces a decohering process which can be strongly constrained by current observations [9]. In this work we correlate the existing limits on gravity-induced decoherence with a lower bound on the scale of quantum gravity. We then examine how this bound can shed light on the nature of TeV-scale gravity.

The SM is based on the gauge group $G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$, with three fermion generations. A single generation consists of five different representations of the gauge group: $Q_L(3, 2, 1/6)$, $U_R(3, 1, 2/3)$, $D_R(3, 1, -1/3)$, $L_L(1, 2, -1/2)$, $E_R(1, 1, -1)$; where the numbers in parenthesis represent the corresponding charges under G_{SM} . The model contains a single higgs boson doublet, $\phi(1/2, 1/2)$, whose vacuum expectation value breaks the gauge symmetry G_{SM} into $SU(3)_C \times U(1)_{\text{EM}}$. An important feature of the SM, which is relevant to spacetime foam interactions discussed here, is the fact that the SM also comprises an accidental global symmetry, $G_{\text{SM}}^{\text{global}} = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$, where $U(1)_B$ is the baryon number symmetry, and $U(1)_{e,\mu,\tau}$ are three lepton flavor symmetries, with total lepton number given by $L = L_e + L_\mu + L_\tau$. It is an accidental symmetry because we do not impose it. It is a consequence of the gauge symmetries and the low energy particle content. It is possible (but not necessary), however, that effective interaction operators induced by the high energy content of the underlying theory may violate sectors of the global symmetry. This violation is already present in the absence of gravity, as has been evidenced by the discovery of neutrino flavor oscillations. If there are additional contributions of this kind from the gravity sector, then black hole-mediated interactions could violate $G_{\text{SM}}^{\text{global}}$ in a distinct manner, as mentioned above and reviewed in what follows.

Measurements of flavor transformations in a neutrino beam can provide a clean and sensitive probe of interactions with the spacetime foam. Without interference from the gravitational sector, oscillations in the neutrino sector provide a pure quantum phenomenon in which the density matrix has the properties of a projection operator, $\text{Tr } \rho^2 = \text{Tr } \rho = 1$. Because black holes do conserve energy, angular momentum (helicity), color and electric charge, any neutrino interacting with the virtual black holes needs to re-emerge as a neutrino. As an example, if spacetime foam black holes do not conserve $U(1)_e \times U(1)_\mu \times U(1)_\tau$, neutrino flavor is randomized by interactions with the virtual black holes. The result of many interactions is to equally populate all three possible flavors.

The ensuing discussion will be framed in the context of $\nu_\mu \leftrightarrow \nu_\tau$ oscillations, and we will comment on other channels after presenting our results. Consider two neutrino states $\nu_1 = (1, 0)^T$ and $\nu_2 = (0, 1)^T$ with masses m_1 and m_2 , and two flavor states $\nu_\mu = (c_\theta, s_\theta)^T$ and $\nu_\tau = (-s_\theta, c_\theta)^T$, where θ is the neutrino mixing angle, $c = \cos$, $s = \sin$, and T denotes the transpose. In the absence of spacetime foam interactions the time evolution equation for the density matrix,

$$\dot{\rho} = -i[H, \rho], \quad (2)$$

is governed (in the mass basis) by the Hamiltonian $H = \frac{1}{2} \text{diag}(-k, +k)$, where $k = \Delta m^2/2E$, $\Delta m^2 = m_2^2 - m_1^2$, and $E (\gg m_{1,2})$ is the neutrino energy (in natural units). The solution $\rho(t)$ of Eq. (2), with initial conditions $\rho(0) = \Pi_{\nu_\mu}$, gives the ν_μ survival probability after propagation

of a distance x ,

$$P(\nu_\mu \rightarrow \nu_\mu) = \text{Tr} [\Pi_{\nu_\mu} \rho(t)] = 1 - s_{2\theta}^2 (1 - \cos kx)/2 . \quad (3)$$

Here, $\Pi_{\nu_\mu} = \nu_\mu \otimes \nu_\mu^\dagger$ is the ν_μ state projector. Transitions from pure to mixed states become possible by addition of a dissipative term $\mathcal{D}[\rho]$ into Eq. (2) [10]

$$\dot{\rho} = -i[H, \rho] - \mathcal{D}[\rho] . \quad (4)$$

Energy conservation, complete positivity (assuring the absence of unphysical effects, such as negative probabilities, when dealing with correlated systems), and monotonic increase in the von-Neumann entropy eventually lead to a modification of Eq. (3) [11],

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - s_{2\theta}^2 (1 - e^{-\gamma x} \cos kx)/2 . \quad (5)$$

Here, γ has dimension of energy, and its inverse defines the typical (coherence) length after which the system gets mixed. Thus, for $\gamma x \sim \mathcal{O}(1)$ one expects significant deviations from Eq. (3). We are interested in dissipative scenarios where decoherence effects vanish in the weak gravitational limit, $M_{\text{QG}} \rightarrow \infty$, and thus we take

$$\gamma = \tilde{\kappa} \left(\frac{E}{\text{GeV}} \right)^n \left(\frac{M_{\text{QG}}}{\text{GeV}} \right)^{-n+1} \text{ GeV} , \quad (6)$$

where $\tilde{\kappa}$ is a dimensionless parameter which by naturalness is expected to be $\mathcal{O}(1)$ and $n \geq 2$.

Atmospheric neutrinos are generated in the decay of pions and kaons resulting from cosmic ray collisions. Most of them are produced in a spherical surface at about 10-20 km above ground level and they proceed towards the earth. Production of muon neutrinos is dominated by the process $\pi^+ \rightarrow \mu^+ \nu_\mu$ (and its charged conjugate). At energies $\lesssim 10$ GeV the muons decay before reaching the surface of the earth, $\mu^+ \rightarrow e^+ \bar{\nu}_\mu \nu_e$. This decay chain then leads to a flavor ratio $\nu_e : \nu_\mu \approx 1 : 2$. The zenith angle distribution observed by Super-Kamiokande shows a clear deficit of upward-going muon neutrinos, which is well explained by two flavor $\nu_\mu \leftrightarrow \nu_\tau$ oscillations, with $\Delta m^2 \simeq 3 \times 10^{-3} \text{ eV}^2$ and $s_{2\theta}^2 \simeq 1$ [12].

A best fit to data collected by the Super-Kamiokande atmospheric neutrino experiment [12], allowing for both oscillation and decoherence yields, for $n = 2$,

$$\tilde{\kappa} \left(\frac{M_{\text{QG}}}{\text{GeV}} \right)^{-1} < 0.9 \times 10^{-27} , \quad (7)$$

at the 90% CL [9]. For larger n , Eq.(7) generalizes to [13]

$$\tilde{\kappa} \left(\frac{M_{\text{QG}}}{\text{GeV}} \right)^{-n+1} < 0.9 \times 10^{-27} . \quad (8)$$

Indeed, this represents a conservative bound. This can be seen from Eq. (6): the analysis of data places bounds on γ , and the neutrino energies are well above 1 GeV. From Eq. (8) it is straightforward to see that, for $\tilde{\kappa} \sim 1$, *the lower limit on the UV cutoff is well beyond the electroweak scale.*

We now comment on other possible oscillation channels. The CCFR detector at Fermilab is sensitive to $\nu_\mu \rightarrow \nu_e$ [14] and $\nu_e \rightarrow \nu_\tau$ [15] flavor transitions. Neutrino energies range from 30 to 600 GeV with a mean of 140 GeV, and their flight lengths vary from 0.9 to 1.4 km. A best fit to the data allowing for both oscillation and decoherence yields

$$\tilde{\kappa} \left(\frac{M_{\text{QG}}}{\text{GeV}} \right)^{-n+1} < 2.0 \times 10^{-24} , \quad (9)$$

at the 99% CL [16]. Moreover, similar constraints on gravity-induced decoherence arise in the quark sector. The persistence of coherence in $K^0\bar{K}^0$ oscillations and neutron interferometry leads to [10]

$$\tilde{\kappa} \left(\frac{M_{\text{QG}}}{\text{GeV}} \right)^{-n+1} < 2.0 \times 10^{-21} . \quad (10)$$

The typical energy in these systems is $E \sim 1$ GeV. In the near future, the possible observation of Galactic anti-neutrino beams [17] at the IceCube facility may provide a major improvement in sensitivity to spacetime foam interactions [18].

In summary, using an existing analysis of atmospheric neutrino data we have shown that if TeV-scale gravity is realized in nature, interactions with virtual black holes are non-dissipative and there is therefore no loss of information. Non-dissipative interactions are expected when gravity is embedded in string theory, so that an S -matrix description is possible. The existence of an S -matrix makes it no longer automatic that, *e.g.*, the B -violating $\bar{q}l$ and B -conserving qq outgoing channels have the same probability, as they would in thermal evaporation. Thus, the problem of avoiding rapid baryon decay or large Majorana neutrino masses is shifted to the examination of symmetries [19] in the underlying string theory which would suppress the appropriate non-renormalizable operators at low energies.

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