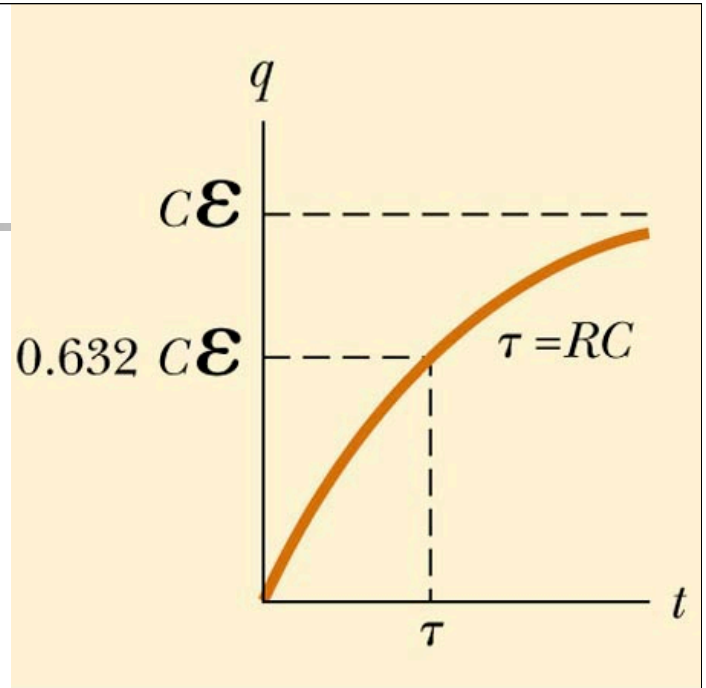
RC summary

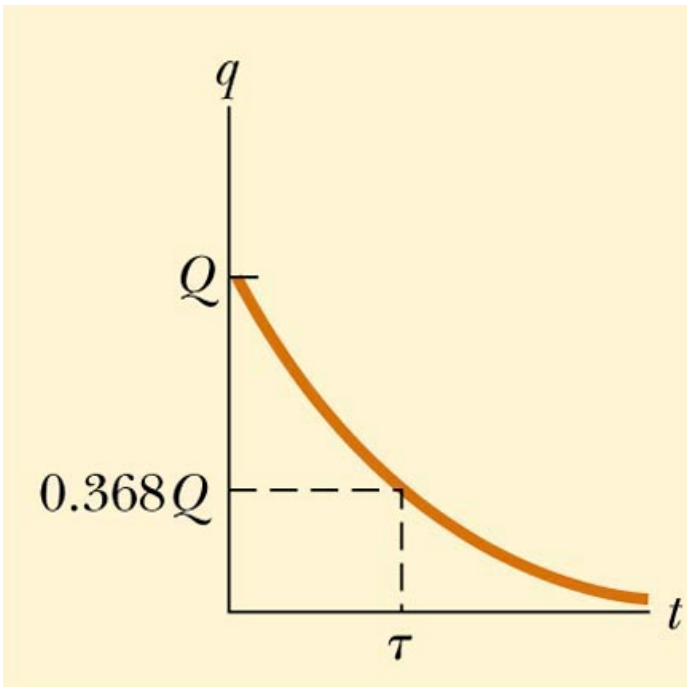
- charge: $\varepsilon = R\frac{dq}{dt} + \frac{q}{C}$
  - $q = C\varepsilon(1 - e^{-t/RC})$
  - The *time constant*, τ=RC

$$I(t) = \frac{\varepsilon}{R}e^{-t/RC}$$



- discharge: q/C = RI I = -dq/dt
  - $q = Qe^{-t/RC}$

$$I(t) = -\frac{dq}{dt} = -\frac{Q}{RC}e^{-t/RC}$$

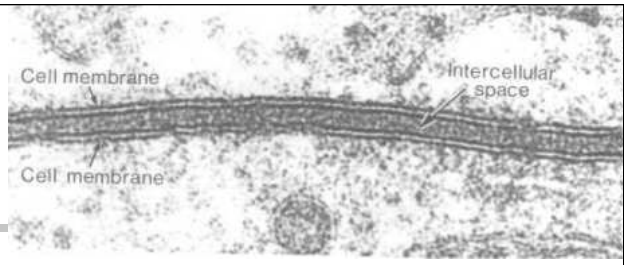


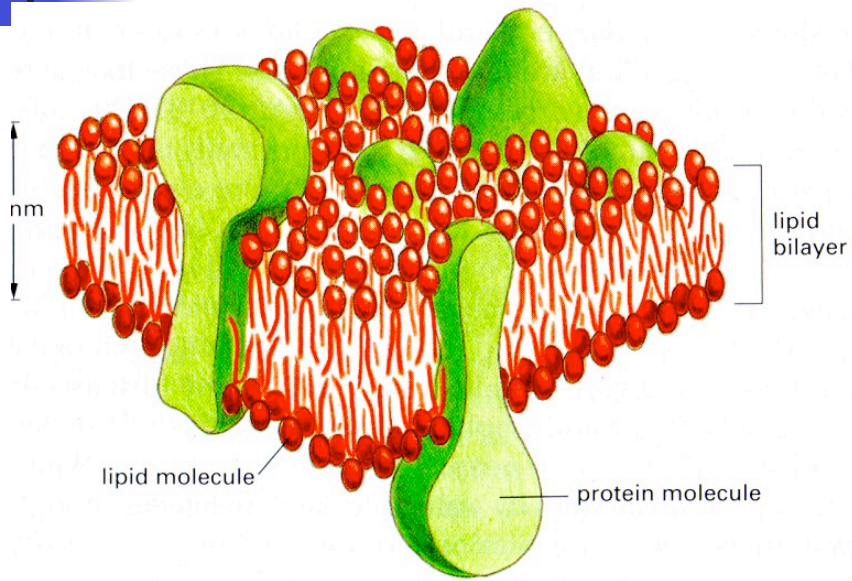
Work to charge a capacitor

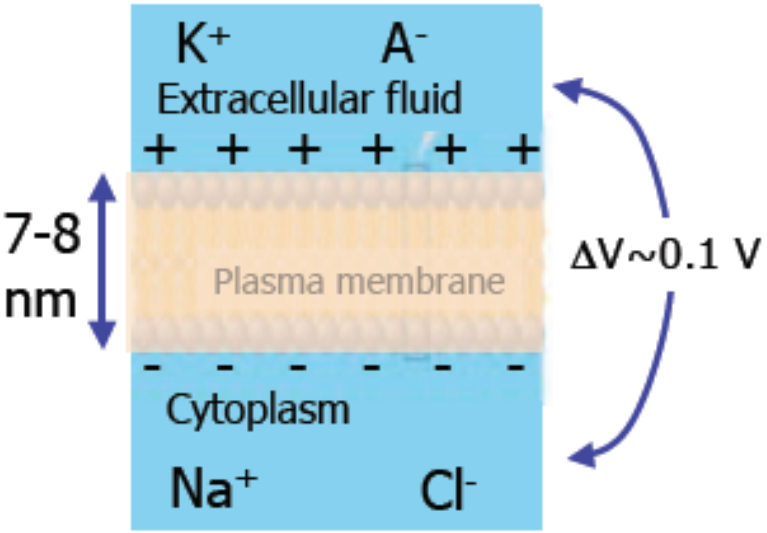
- The final charge is Q = cε
- W = Qε=Cε^2
- 1/2 of this work is energy stored in the capacitor U = 1/2Cε^2
- energy dissipated in R

$$W_R = \int_0^\infty R \frac{\varepsilon^2}{R^2} e^{-2t/RC} dt = \frac{\varepsilon^2}{R} \frac{RC}{2} = \frac{1}{2} C\varepsilon^2$$

Membranes as capacitors

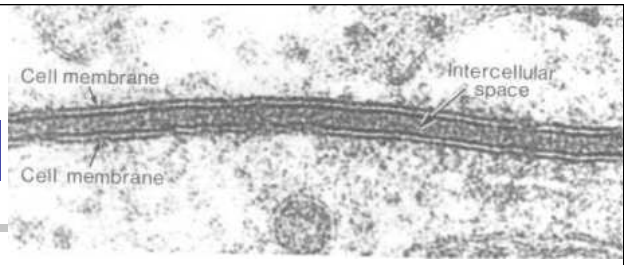




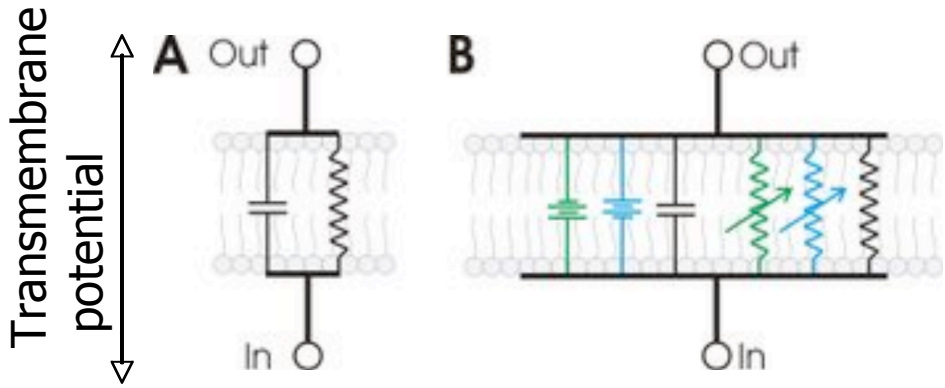


- Lipid bilayers of cell membranes are like plates of a capacitor made of polar lipid heads separated by a dielectric layer of hydrocarbon tails
- Due to the ion distribution inside and outside the cell there is a potential difference called resting potential
- ion channels allow current flow across the membrane

Membrane Electrical Model



- **specific resistance of membranes: $R = \rho L/A \Rightarrow R\,A = \rho L = 10\text{-}10^6\ \Omega\ cm^2$**
  capacitance/area $C/A = 1\ \mu F/cm^2$
- The simplest model for a biological membrane is an RC circuit
- When S is closed (= membrane channel opens) ⇒discharge of C
- RC time constants range from **10 μs to 1 s = (RA)(C/A)**



A) Basic RC circuit for a membrane superimposed to the bilayer image and a membrane with ion channels

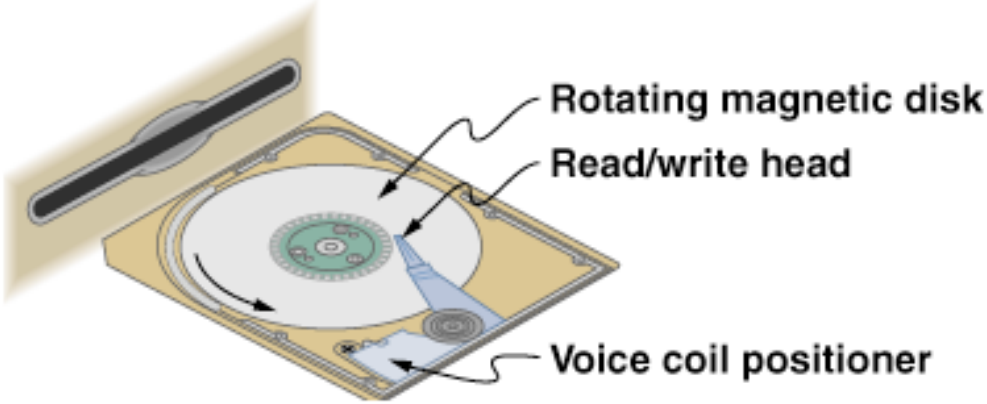
B) Cell membrane with ion channels

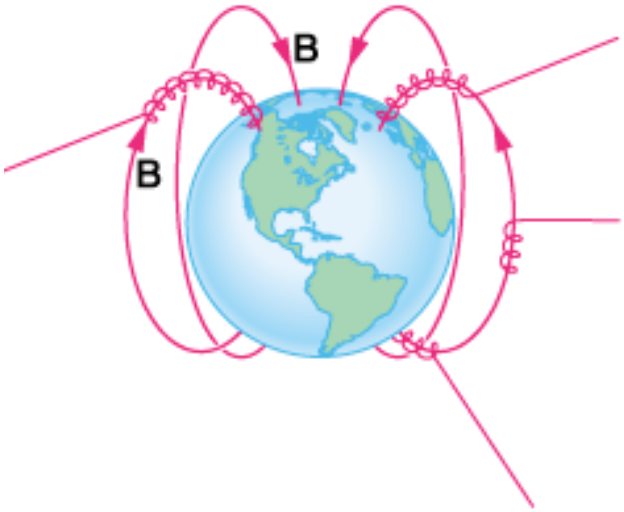
Magnetic Fields

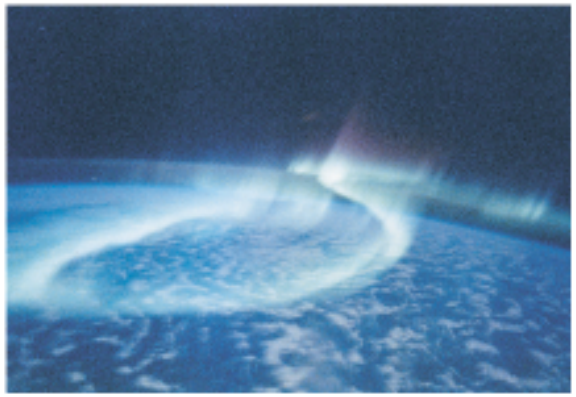
This Lecture (Ch 26 T&M):

- Magnetic Fields and Magnets
- Motion of charges in Magnetic Fields
- Force on a wire with current
- Torques on Current loops

Magnetic Fields in ordinary life

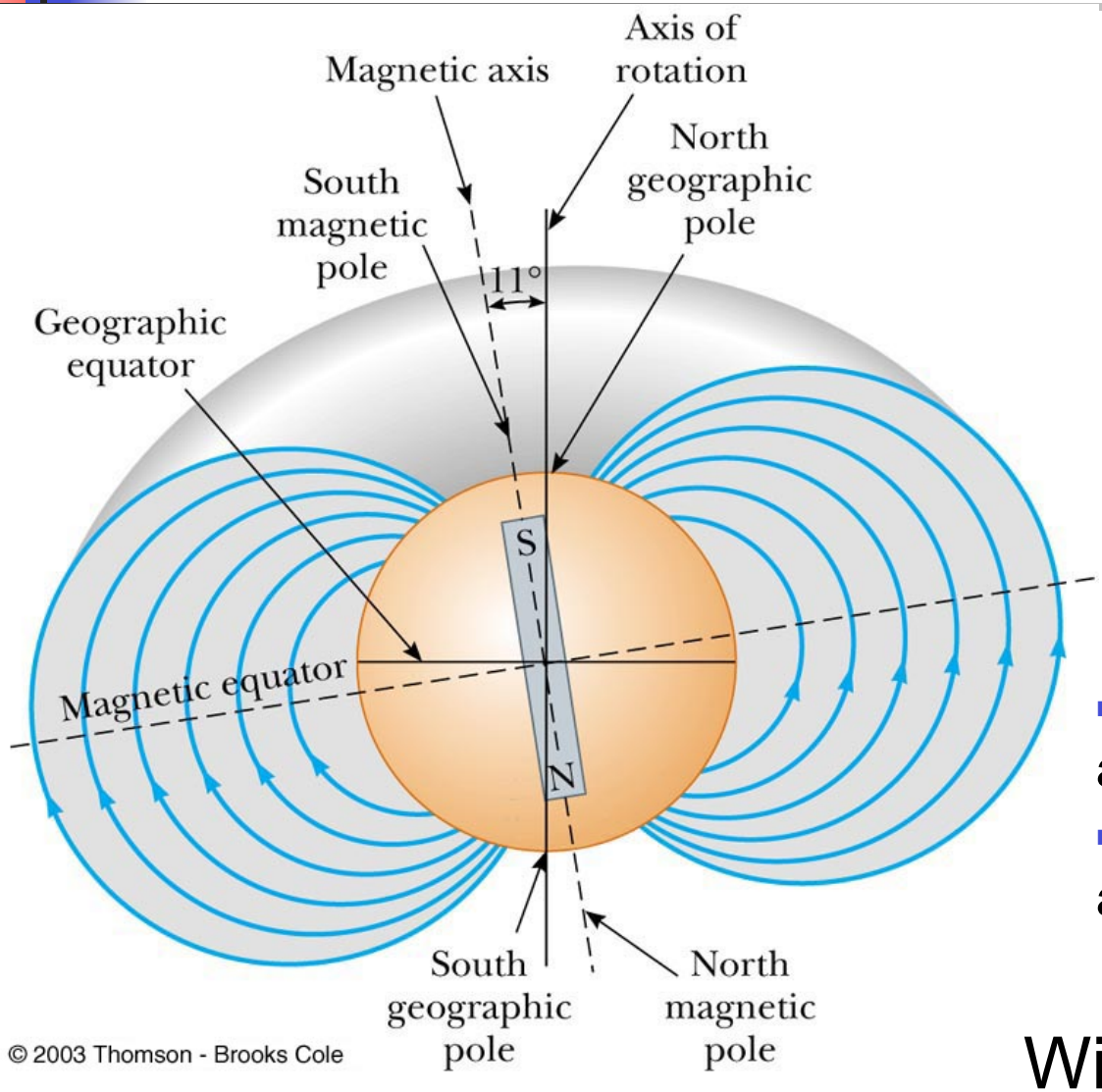






Aurora Borealis

Earth's A Magnet!



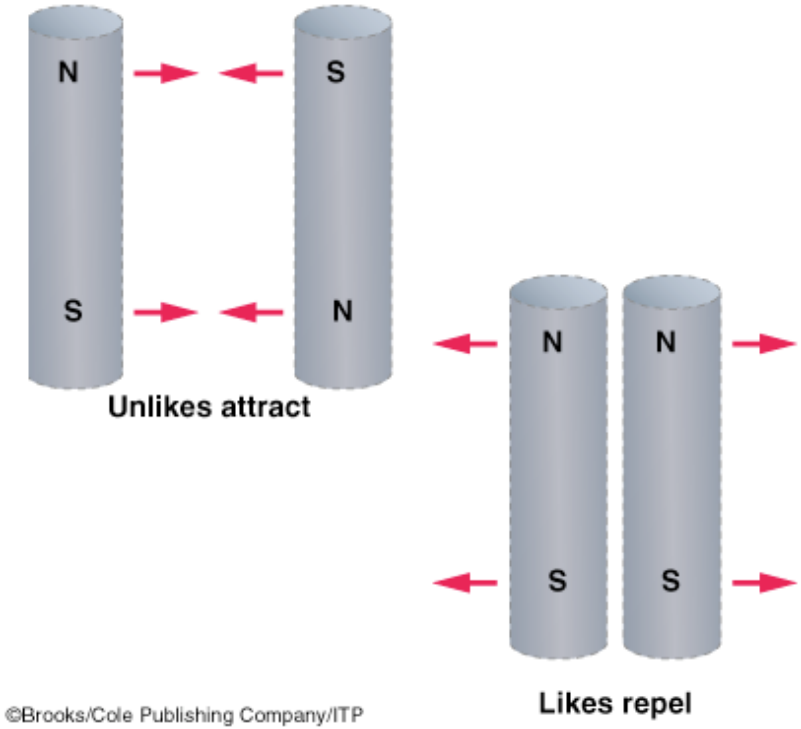
North magnetic pole is about half way around the Earth (πR_E) from the North geographic pole

- N geographic pole almost at magnetic S pole
- S geographic pole almost at magnetic N pole

William Gilbert (1600)

Magnets

- 13th century BC: Chinese already used a compass with a magnetic needle (Arabic or Indian origin?)
- 800 BC: Greeks discovered magnetite (Fe_3O_4)
  - ***Like poles repel each other***
    - N-N or S-S
  - ***Unlike poles attract each other***
    - N-S



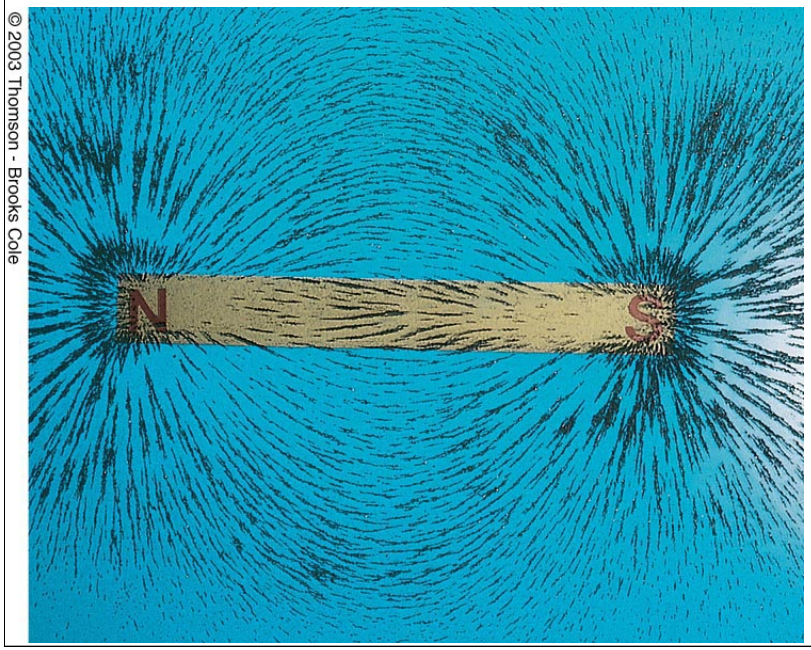
Sources of magnetic field

- Magnetic Fields are created by moving electric charge!
- 1819 Hans Christian Oersted
  - Discovered the relationship between electricity and magnetism
  - An electric current in a wire deflected a nearby compass needle
- 1820's
  - Faraday and Henry
    - Further connections between electricity and magnetism
    - A changing magnetic field creates an electric field
  - Maxwell
    - A changing electric field produces a magnetic field

Magnetic Fields

- A vector quantity (**B**)
- compass needle traces B field lines and points towards N
- The lines outside the magnet point from N to S (do not stop because there is no monopole)

Iron filings show the pattern of the electric field lines

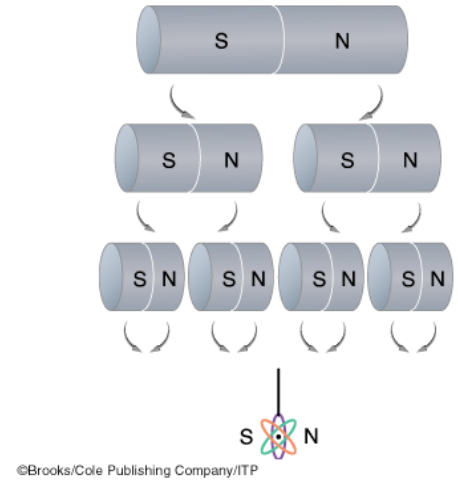


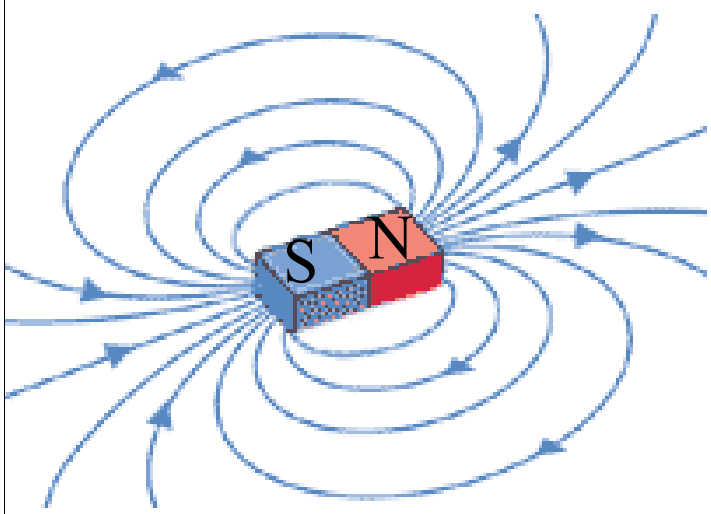
- B-Field lines as E-lines
  - Density gives strength
  - Arrow gives direction
    - Leave +, North
    - Enter -, South
- Differences
  - Start/Stop on electric charge
  - No Magnetic Charge ⇒ lines are continuous!

Let's Break A Magnet!

- Magnetic poles are always found in pairs!

A monopole has never been observed (but…)!

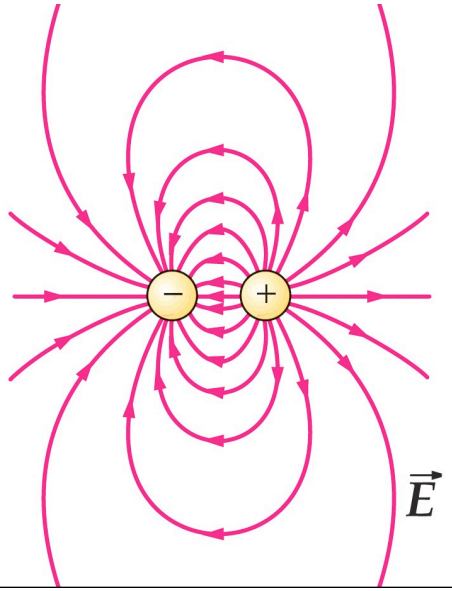




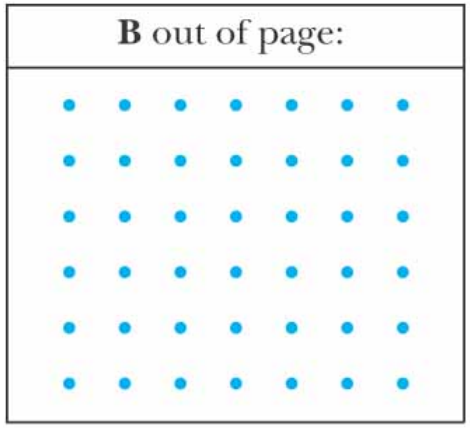
Only in the region between the 2 charges E-lines differ from B ones

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

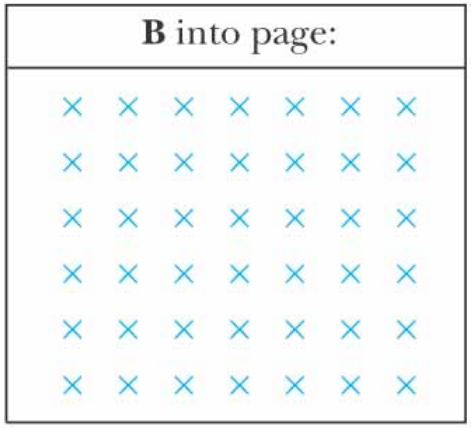
Electric dipole



B-field representation



(a)

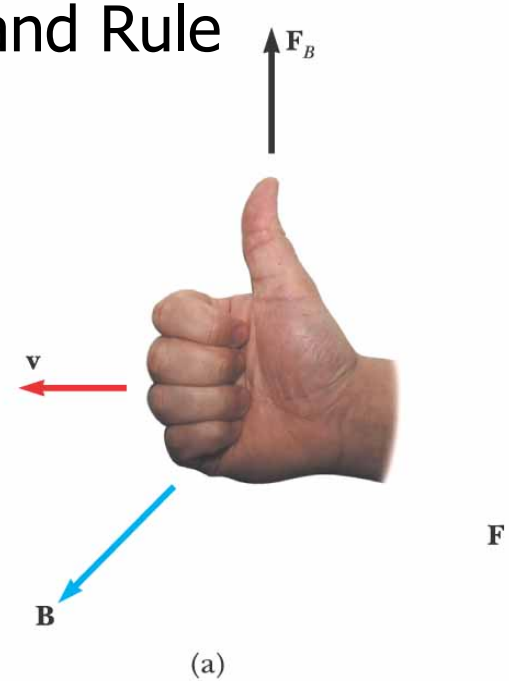


(b)

$\mathbf{F}_B$ on a Charge Moving in a Magnetic

$$\mathbf{F}_B = q\,\mathbf{v} \times \mathbf{B}$$

Right Hand Rule



- **v** is the velocity of the moving charge q

- SI unit of magnetic field: **tesla** (T)

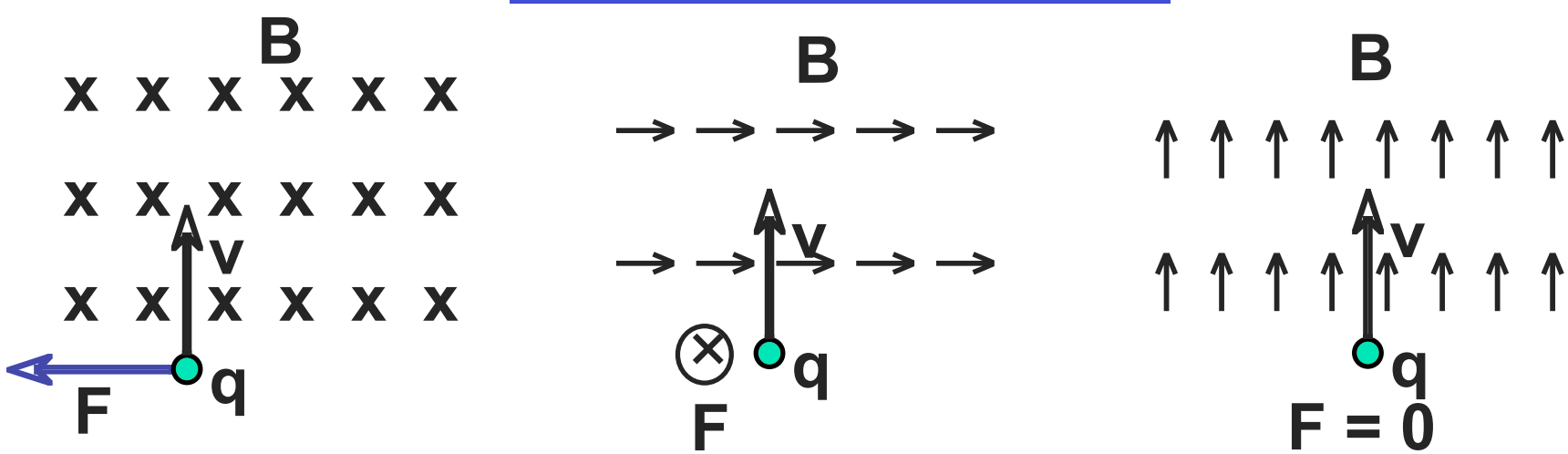
$$T = \frac{Wb}{m^2} = \frac{N}{C\cdot(m/s)} = \frac{N}{A\cdot m}$$

- CGS unit: *gauss* (G):
- 1 T = 10^4 G (Earth surface 0.5 G)

- The magnetic force does no work when a particle is displaced (force perpendicular to displacement)
- the field can alter the *direction* of the velocity, but not the speed or the kinetic energy

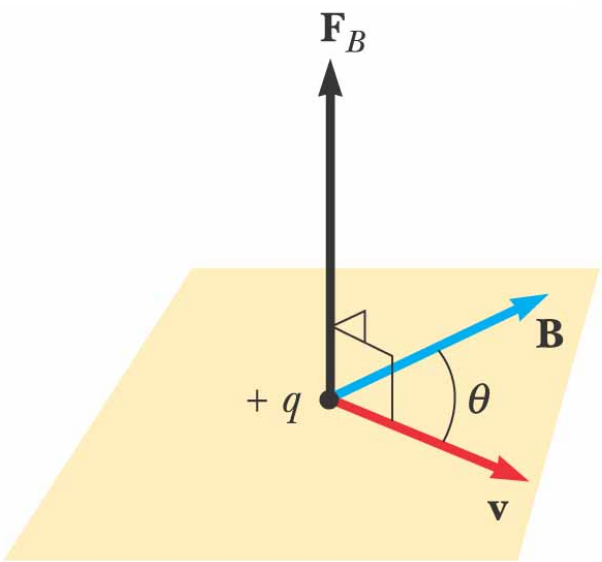
Lorentz Force

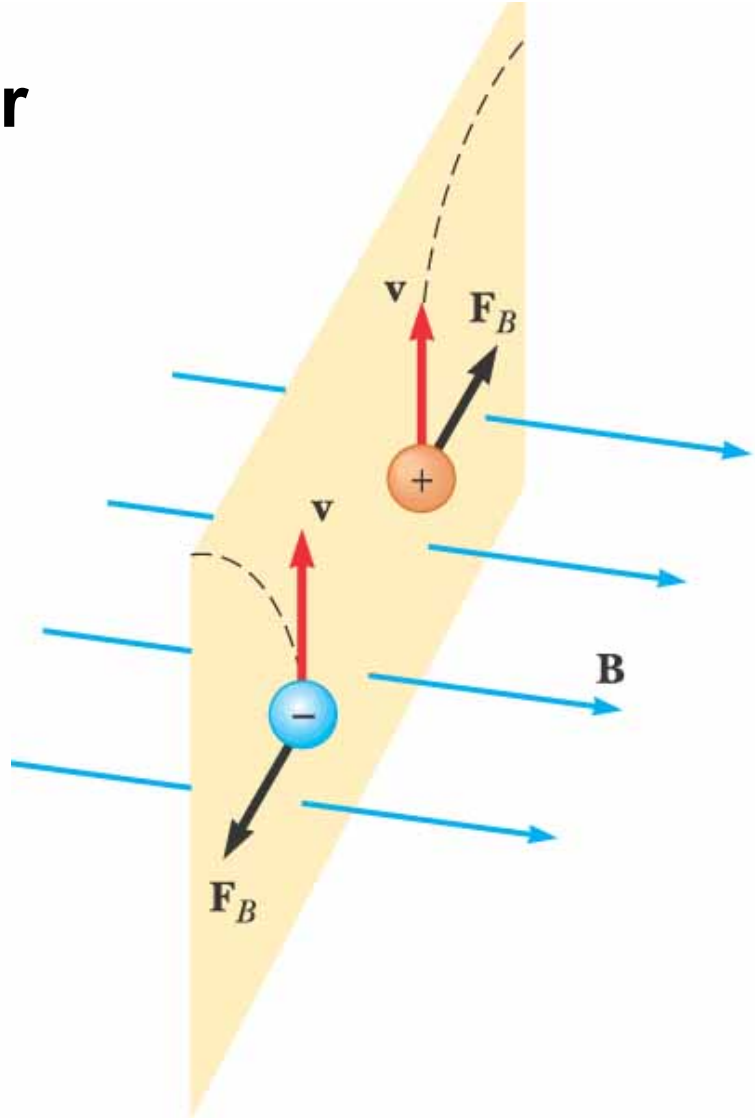
$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$



Effect of uniform magnetic field on a charged particle in motion

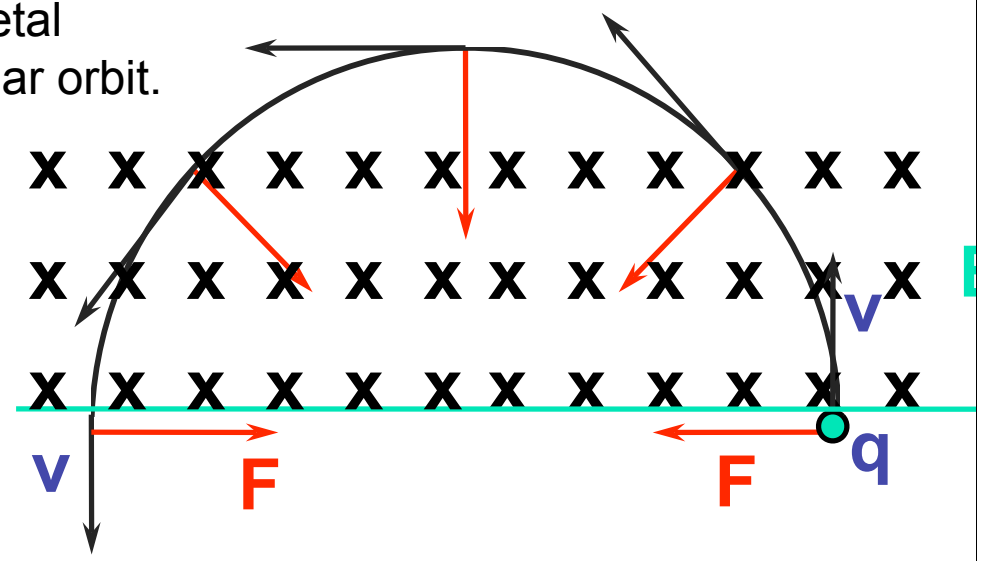
- If particle is moving: force exerted **perpendicular** to *both* field and velocity





Trajectory in Constant B orthogonal to plane of motion

Path is a circle. F/m is the centripetal acceleration of the particle in circular orbit.



$$F = ma$$

$$qvB = m\frac{v^2}{R} \Rightarrow R = \frac{mv}{qB}$$

$$\omega = \frac{qB}{R}$$

The cyclotron angular frequency does not depend on v