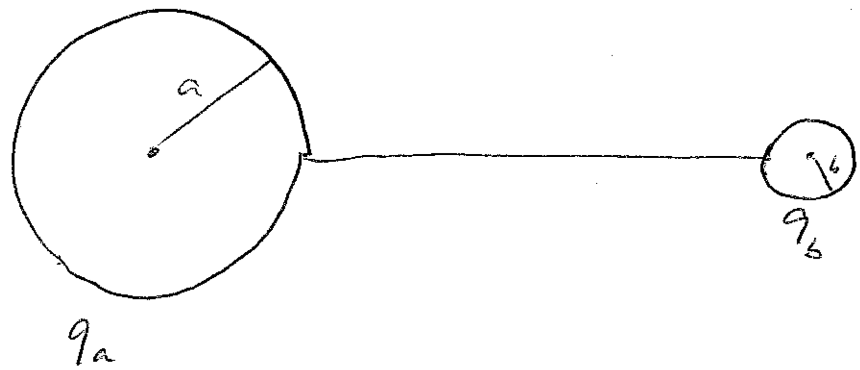
Lecture 27

Conductor



$$V = k\,\frac{q_a}{a} = k\,\frac{q_b}{b}$$

$$\frac{q_a}{q_b} = \frac{a}{b}$$

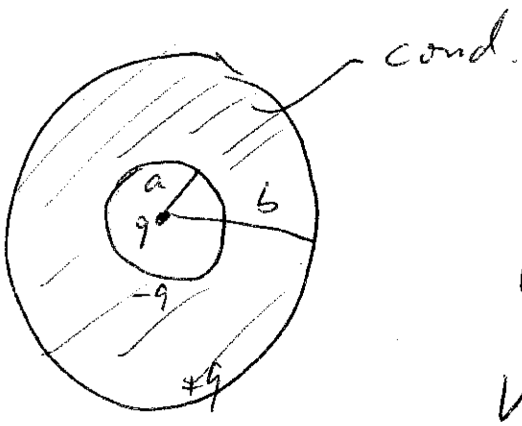
E ?

Approx. (far apart):

$$E_a = k\,\frac{q_a}{a^2} \qquad E_b$$

$$\frac{E_a}{E_b} = \frac{q_a\ b^2}{a^2 \cdot q_b} = \frac{b}{a}$$

$$E_b = \frac{a}{b} \cdot E_a$$



$V = k\frac{q}{r}$, $r > b$

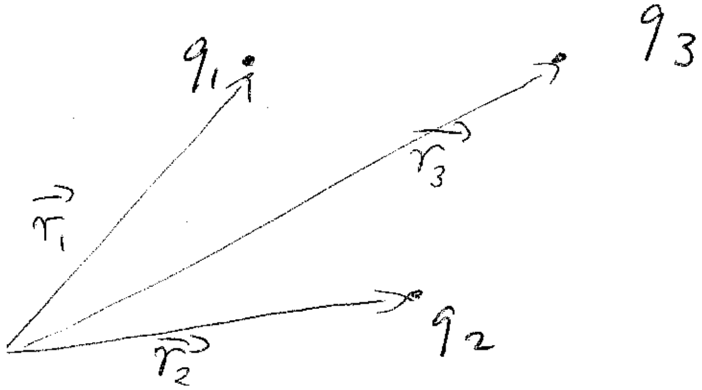
$V = k\frac{q}{b}$, $a < r < b$

$V = k\frac{q}{r} + \text{const} \qquad r < a$

$V(a) = k\frac{q}{a} + \text{const} = k\frac{q}{b}$

$\rightarrow V = k\frac{q}{r} + k\left(\frac{q}{b} - \frac{q}{a}\right) \qquad r < a$

Electrostatic potential energy



$$U = \underbrace{W_{21}}_{\text{Bring } q_2} + \underbrace{W_{31} + W_{32}}_{\text{Bring in } q_3}$$

$$= k\, q_2 \underbrace{V_1(\vec{r}_2)}_{\frac{k q_1}{r_{1,2}}} + q_3 \underbrace{V_1(\vec{r}_3)}_{\frac{k q_1}{r_{1,3}}} + q_3 \cdot V_2(\vec{r}_3)$$

$$= \cancel{k}$$

$$= k \frac{q_2 q_1}{r_{1,2}} + k \frac{q_3 q_1}{r_{1,3}} + k \frac{q_3 q_2}{r_{2,3}} = \sum_{i>j} k \frac{q_i q_j}{r_{j,i}}$$

$$= k \frac{q_1 q_2}{r_{2,1}} + \ldots = \sum_{i<j} \frac{q_i q_j}{r_{j,i}}$$

We can reverse the order ~~and~~ of assembly
$q_1\ q_2\ q_3 \rightarrow q_3\ q_2\ q_1$

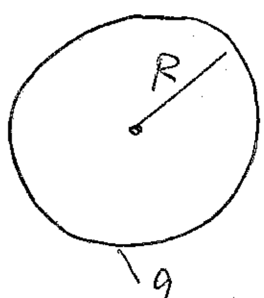
$$= \frac{1}{2} \sum_{i \neq j} \frac{q_i q_j}{r_{i,j}} = \frac{1}{2} \sum_i q_i \sum_{j \neq i} \frac{q_j}{r_{i,j}} = \frac{1}{2} \sum_i q_i V_i$$

$$V_i = \sum_{j \neq i} \frac{q_j}{r_{i,j}}$$

= potential at $\vec{r}_i$ due to all othe charges $q_j(\vec{r}_j)$

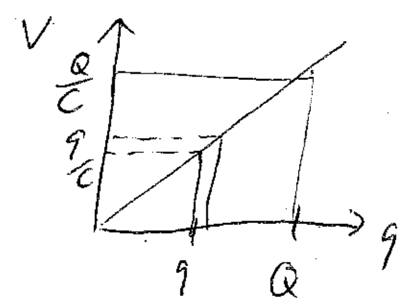
Continuous q-distribution $\rho = \rho(\vec{r})$ $\left[\frac{C}{m^3}\right]$

Example: cond. sphere



$$V = \frac{k\,q}{R}$$

$V(\infty) = 0$



Add dq:

$$dU = V\,dq = k\,\frac{q}{R}\,dq$$

$$U = \frac{1}{R}\int_0^Q q\,dq = \frac{1}{2}\,\frac{Q^2}{R} = \frac{1}{2}\,Q\,V$$

true for any shape of conductor

Capacitance

Cond. Sphere: $V = k \dfrac{Q}{R}$

$\rightarrow$ constant $= \dfrac{R}{k} = \dfrac{Q}{V}$ characteristic property of sphere = capacitance

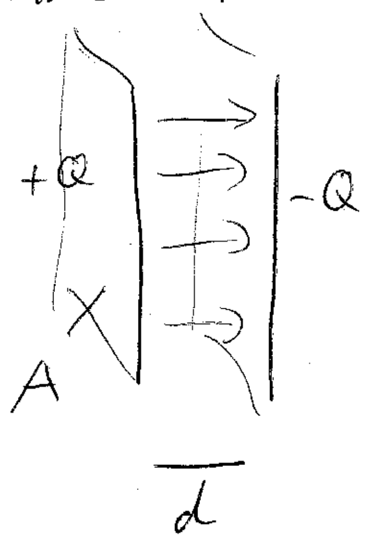
$$\boxed{C_{sphere} = \frac{R}{k} = 4\pi \varepsilon_0 R}$$

General

$$\boxed{C = \frac{Q}{V}}$$ capacitance

$$1\ F = 1\ \frac{C}{V}$$

Parallel Plate:



$V = E \cdot d =$

One Inf. plate $E = \sigma / 2\varepsilon_0$

$= \dfrac{\sigma}{\varepsilon_0} \cdot d = \dfrac{Qd}{\varepsilon_0 A}$

$$\boxed{C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}}$$

Assume $A \gg d$

$\sigma = \dfrac{Q}{A}$

Stored energy

$$dU = V\,dq = \frac{q}{C}\,dq$$

$$U = \frac{1}{2}\,\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

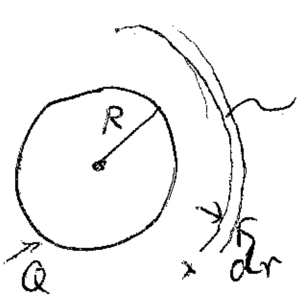
For parallel plate capacitor

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)\cdot(Ed)^2$$

$$= \underbrace{\frac{1}{2}\varepsilon_0 E^2}_{\frac{\text{energy}}{\text{Volume}}} \cdot \underbrace{(A\cdot d)}_{\text{Volume}}$$

$$\boxed{u_e = \frac{1}{2}\varepsilon_0 E^2}$$ energy density of any E-field!

→ Spherical cond., again:



$$dV = 4\pi r^2\,dr \qquad E = \begin{cases} \frac{kQ}{r^2} & r>R \\ 0 & r<R \end{cases}$$

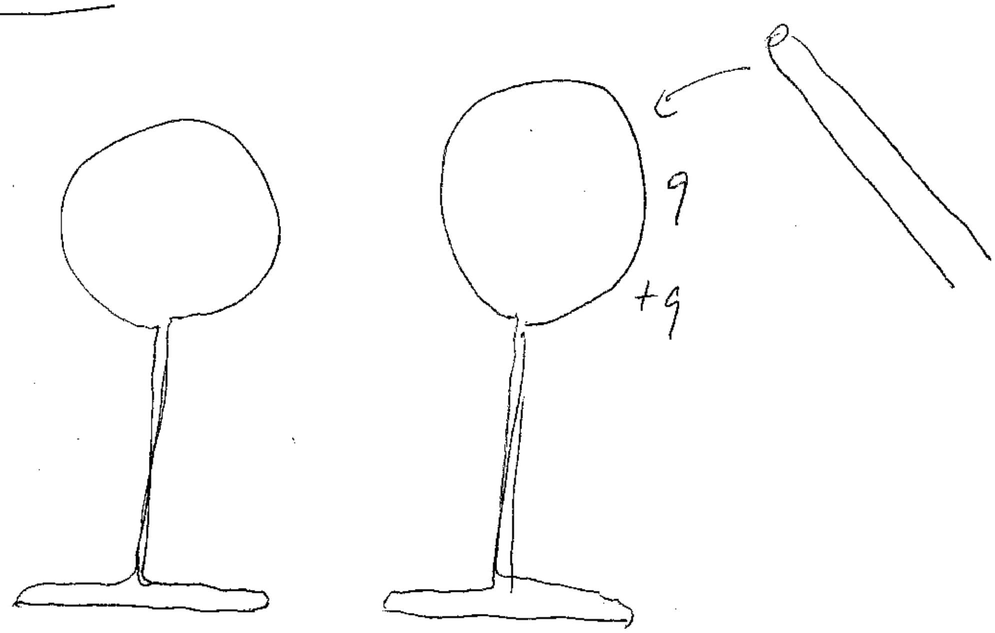
$$U = \int \frac{1}{2}\varepsilon_0 E^2\,dV$$

$$= \frac{1}{2}\varepsilon_0 \int_{r=R}^{\infty} E^2\,4\pi r^2\,dr = \cancel{\tfrac{1}{2}}\,4\pi\varepsilon_0 (kQ)^2 \int \frac{1}{r^4}\,r^2\,dr$$

$$= \frac{1}{2}\left(\frac{kQ^2}{R}\right) = \frac{1}{2}\,\frac{Q^2}{C} \quad ✓$$

Demos: $\overset{\text{charging}}{\text{cond. spheres}}$

a)

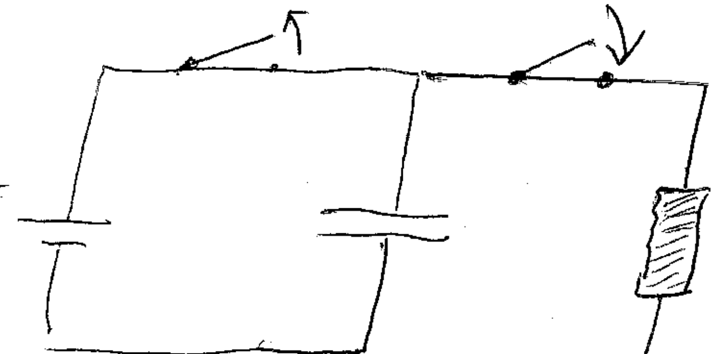


b) Stored energy in capacitor

Discharging a big capacitor through aluminum foil

6000 V

+



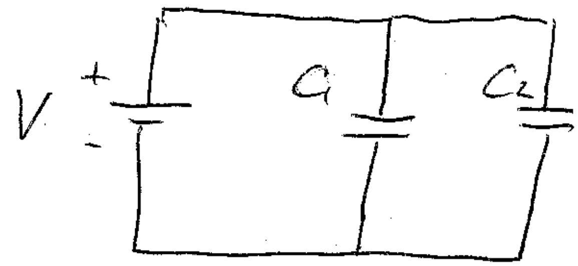
Al-foil = resistor

Bang

Foil is vaporized

Circuits with capacitors

1.) Parallel



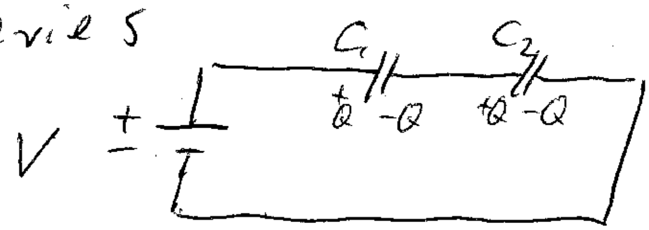
$$Q_1 = C_1 V$$
$$Q_2 = C_2 V$$

$$Q = Q_1 + Q_2 = (C_1 + C_2)\cdot V$$

$$C_{total} = \frac{Q}{V} = C_1 + C_2$$

$$\boxed{C = C_1 + C_2}$$

2.) Series

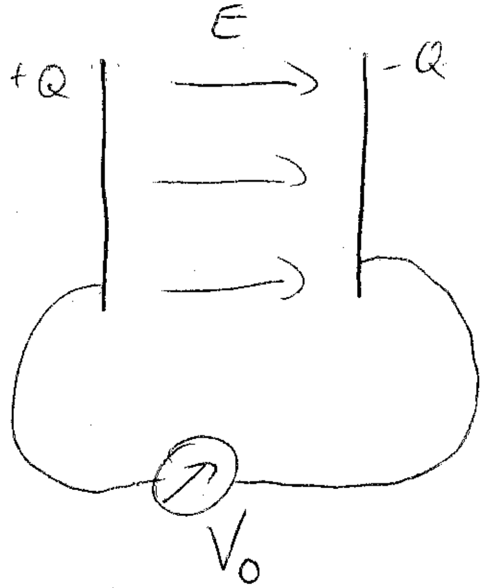


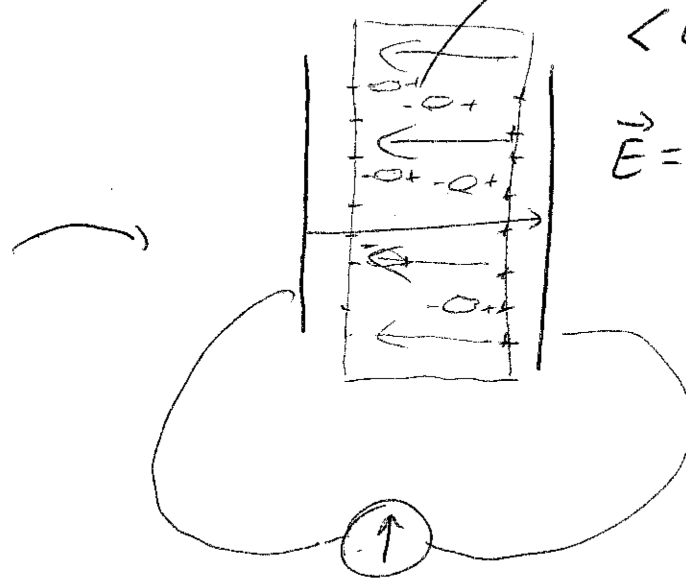
$$\left.\begin{array}{l} V = V_1 + V_2 \\ Q = Q_1 = Q_2 \end{array}\right\} = \left\{\frac{Q}{C_1} + \frac{Q}{C_2}\right\}$$

$$C = \frac{Q}{V} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Dielectrics:





$E = E_{ext} + E_{dip}$

$< E_{ext}$

$\vec{E} = \dfrac{E_{ext}}{\varkappa}$

$$V = \frac{V_0}{\varkappa}$$

Molecules are dipoles

$\rightarrow$ net surface charge

$$E = \frac{E_0}{\varkappa} \qquad V = E d = \frac{V_0}{\varkappa}$$

$$\rightarrow \quad C = \frac{Q}{V} > C_0$$

smaller