LECTURE 26

Quick review:

Ch 21:

$$\vec{r}_{1,2} = \vec{r}_2 - \vec{r}_1$$

$$\vec{F}_{e\,12} = k_e \cdot \frac{q_1 q_2}{r^2}\,\hat{r}_{12}$$

$$\vec{E} = \vec{F}/q_0$$

E from discr. charge distr.

$$\vec{E}_{i,P} = \sum \vec{E}_i = \sum \frac{k q_i}{r_{i,P}^2}\,\hat{r}_{i,P}$$

= superposition

Ch. 22:

$$\vec{E} = \int_V \frac{k\,dq}{r^2}\,\hat{r}$$

$$\begin{aligned} dq &= \rho\, dV && \text{volume} \\ &= \sigma\, dA && \text{surf.} \\ &= \lambda\, dl && \text{lin} \end{aligned}$$

Gauss Law:

$$\Phi_{net} = \boxed{\int_S E_n\, dA = 4\pi k Q_{inside}}$$

$$Q_{inside} = \int \rho\, dV$$

espec. useful for ~~spherical symmetry~~
simple geometries

(for complex geometry use numerical integr. w. computer)

Ch 23: El. Potential $\leftrightarrow$ pot. energy

El. potential

$$U \quad \leftrightarrow \quad V$$

$$dU = q_0 \cdot dV$$

$$dV = \frac{dU}{q_0} = -\vec{E}\,d\vec{l}$$

Coulomb potential

$$V = \frac{kq}{r} \qquad \left(\int \vec{E}\,d\vec{l} = \int_1^2 \frac{kq}{r^2}\right)$$

$$= \int \frac{k\,dq}{r}$$

$$dq = \rho\, dV$$

$\sigma\, dA$ ← disk, finite / infinite

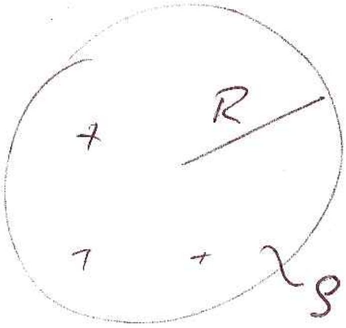
$\lambda\, dl$ ← ex. ring

Useful; often much easier

E from V

$$\vec{E} = -\vec{\nabla} V$$

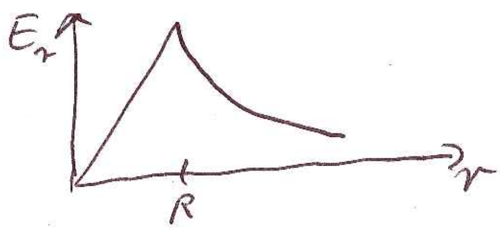
Uniformly charged sphere



$$E_r = k\,\frac{Q}{R^3}\,r \qquad r<R$$

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

Gauss (Ch. 22)



Find V from $\vec{E}$:

$$dV = -\vec{E}\,d\vec{l}$$

a) outside $V = k\,\frac{Q}{r}$

b) inside
$$V = -\int_{\infty}^{r} k\,\frac{Q}{R^3}\,r\,dr \qquad r \le R$$

$$= k\,\frac{Q}{R} - \int_{R}^{r_i} \frac{kQ}{R^3}\,r\,dr$$

$$= k\,\frac{Q}{R} - \frac{1}{2}\,\frac{kQ}{R^3}\cdot\left(r_i^2 - R^2\right)$$

$$= \frac{1}{2}\,k\,\frac{Q}{R}\left(3 - \frac{r^2}{R^2}\right) \qquad r \le R$$

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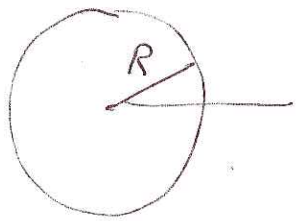
Spherical Shell of Charge:



Outside:

$$\vec{E} = \frac{kQ}{r^2}\hat{r} \qquad r > R$$

(Gauss: $E \cdot 4\pi r^2 = 4\pi k Q_{inside}$)

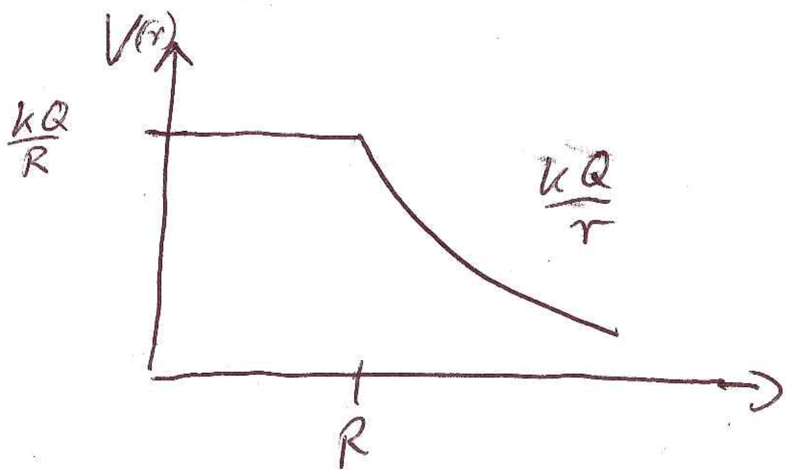
$$\cancel{dV=}\quad V = -\int_{\infty}^{r} \vec{E}\,\vec{dl} = -\int_{\infty}^{r} \frac{kQ}{r^2}\,dr = \frac{kQ}{r}$$

$r > R$

Inside: $\vec{E} = 0$ for $r < R$ (Faraday cage)

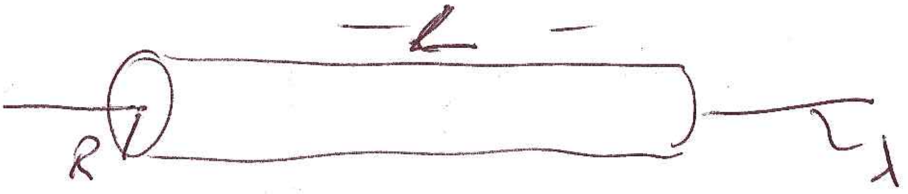
$$\rightarrow dV = \underset{=0}{\underset{\uparrow}{\vec{E}}}\,\vec{dl} = 0 \qquad \text{for } r < R$$

$$V = \text{const} = V(R) = k\frac{Q}{r}$$



Inf. line charge

Symmetry $\rightarrow$ use E, Gauss law then construct V



$$\Phi = \int E \, dA = 4\pi k Q_{inside}$$

$$E \cdot \cancel{2}\cancel{\pi} R \cdot \cancel{L} = 2\cancel{4}\cancel{\pi} k \lambda \cdot \cancel{L}$$

$$E = 2k\lambda / R$$

$$\Delta V = -\int_{R_{ref}}^{R_P} E \, dR = -2k\lambda \int \frac{dR}{R}$$

$$= 2k\lambda \ln \frac{R}{R_{ref}}$$

(Direct integr. ~~con~~ti doesn't work)