Lecture ~~24~~ 25

03/23/07

Friday

Chap 23 - Electric Potential

$W = -\int_{x_s}^{x} kx\,dx$

$F = -kx \qquad W = -\frac{1}{2}kx^2$

$\oplus \qquad \oplus \rightarrow$

$F = qE$

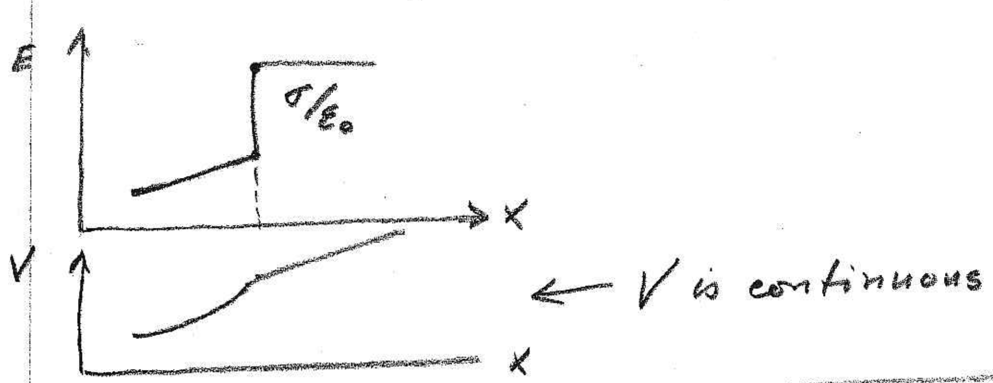
$dU = -dW = -q\vec{E}\cdot d\vec{r}$

$dV = \frac{dU}{q} = -E\cdot dr$ potential per unit charge

$\Delta V = V_b - V_a = \frac{\Delta U}{q} = -\int_a^b E\cdot dl = \frac{\text{joules}}{\text{Coulomb}} = \text{Volt}$

V is continuous

E can jump by σ/ϵ_0



Conservation of energy

$K = \frac{1}{2}mv^2 =$ kinetic energy

$W =$ work done on system

$\frac{1}{2}mv_1^2 + W = \frac{1}{2}mv_2^2$

$K_1 + \int_1^2 F\cdot dx = K_2$

$K_1 + \int_1^{x_s} + \int_{x_s}^2 = K_2$

$U_1 = -\int_{x_s}^{1} \qquad = U_2 - \int_{x_s}^{2}$

$F = -dU/dx \Leftarrow U_a = -\int_{x_s}^{a} F\,dx$

$\boxed{K_1 + U_1 = K_2 + U_2}$ cons of energy

$$\Delta V = \frac{\Delta U}{q} = \frac{\text{joules}}{\text{coulomb}}$$

$$\Delta V = -\bar{E} \cdot d\bar{r} = \vec{\nabla} V \cdot d\vec{r} = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy \qquad \text{(2-dimension)}$$

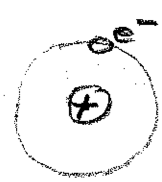
$$= \left(\hat{i} \frac{\partial V}{\partial x} + \hat{j} \frac{\partial V}{\partial y}\right) \cdot \left(\hat{i} dx + \hat{j} dy\right)$$

$$\nabla V \quad \cdot \quad d\bar{r}$$

then $E = -\vec{\nabla} V = -i \frac{\partial V}{\partial x} - j \frac{\partial V}{\partial y}$

unit $= -\frac{\text{volts}}{\text{meter}}$ measure E in $\frac{\text{Volts}}{\text{meter}} = \frac{N}{C}$

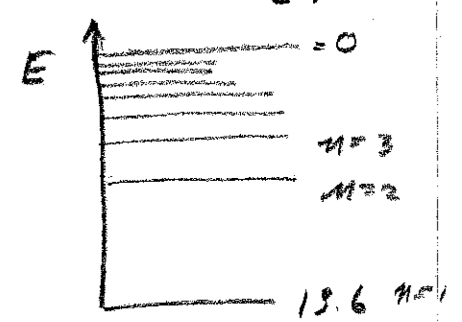
$U = qV$

H-atom 

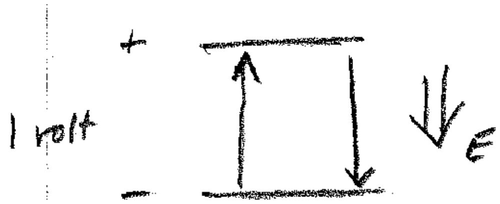
remove e^- to infinity
attractive force - so work is
required ($W = 13.6$ eV)

$$W = -U = -qV = -eV = -(-1.6 \times 10^{-19} C)(1 \,^J/_C)$$

$$= 1.6 \times 10^{-19} J = 1 \, eV$$



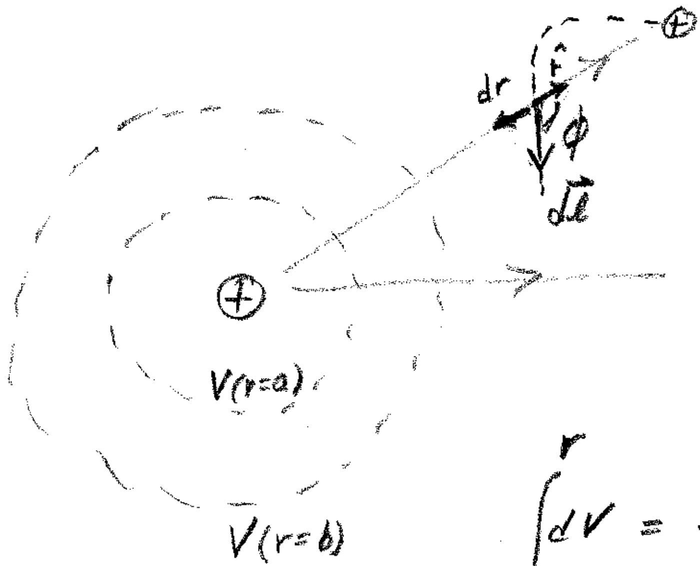
$E_n = -\frac{13.6}{n^2}$



electron gets
1 eV of Kinetic
energy
$\Delta U = -1$

external agent
must supply 1 eV
of energy to electron
increase $\Delta U = +1$

Potential



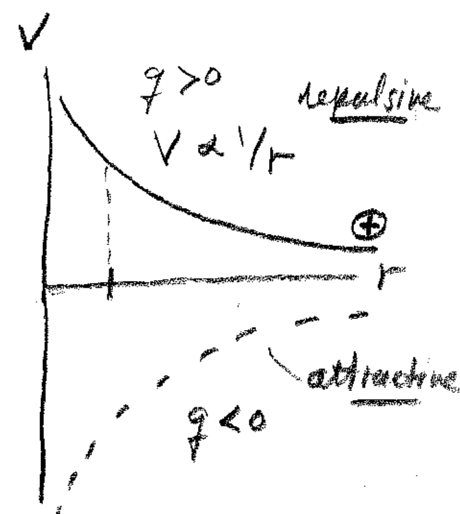
$$E = \frac{kq}{r^2}\hat{r}$$

$$dV = -\vec{E}\cdot\vec{dl} = -\frac{kq}{r^2}\hat{r}\cdot\vec{dl}$$

$$dV = -\frac{kq}{r^2}\,dr$$

$$\int_{r_0}^{r} dV = -\int_{r_0}^{r} E\cdot dl = -\int_{r_0}^{r}\frac{kq}{r^2}\,dr = \frac{kq}{r}\Big|_{r_0}^{r}$$

$$\Delta V = \frac{kq}{r} - \frac{kq}{r_0}$$



define $V = 0$ at $r_0 = \infty$

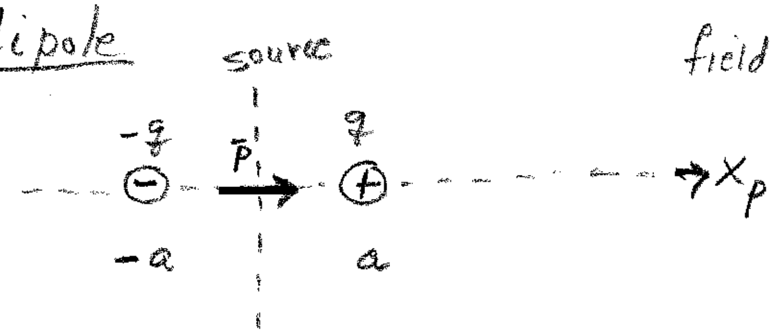
$$V = \frac{kq}{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

Potential of ⊕ test charge q_0

$$U = \underbrace{q_0}_{\text{scale factor}} V = k\frac{q_0 q}{r}$$

scale factor — same graph.

<u>dipole</u>



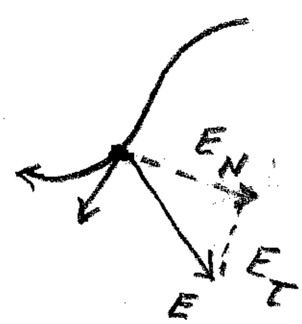
$p = 2qa$

$$V = \frac{-kq}{x+a} + \frac{kq}{x-a} = \frac{2kqa}{x^2-a^2} = \frac{kp}{x^2\left(1-x^2/a^2\right)} \rightarrow \frac{kp}{x^2}$$

$$E_x = -\frac{\partial V}{\partial x} = \frac{2kp}{x^3} = \frac{4kqa}{x^3}\,\hat{\imath}$$ (see page 665)

Fields from Potentials

$$dV = -\vec{E}\cdot d\vec{l} = -E\cos\theta\, dl$$
$$= -E_T\, dl$$



$$E_T = -\frac{dV}{dl}$$ ~ directional derivative in calculus

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy = \left(\hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y}\right)\cdot\left(\hat{\imath}\,dx + \hat{\jmath}\,dy\right)$$

$$dV = \nabla V \cdot dl = -E\cdot dl$$

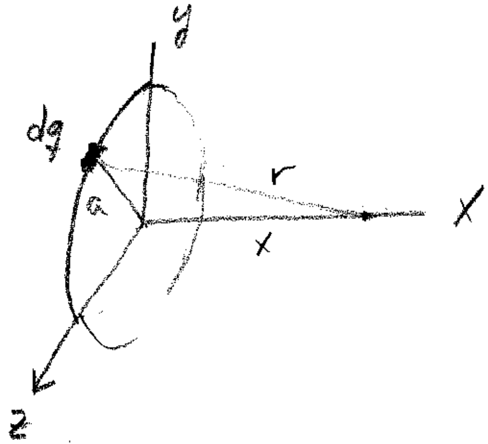
$$E = -\nabla V$$

$$E = -\vec{\nabla} V \qquad E_x = -\frac{\partial V}{\partial x}, \text{ etc}$$

<u>charged ring</u>

$$dV = \frac{k\,dq}{r}$$

$$V = \int_0^Q \frac{k\,dq}{(x^2+a^2)^{1/2}} = \frac{kQ}{\sqrt{x^2+a^2}}$$



$$= \frac{kQ}{|x|\left(1+\frac{a^2}{x^2}\right)^{1/2}}$$

$$\cong \frac{kQ}{|x|}\left(1-\frac{1}{2}\frac{a^2}{x^2}\right)$$

$$= \frac{kQ}{|x|} - \frac{\frac{1}{2}kQa^2}{x^3}$$

monopole pt charge $E \sim 1/r^2$ — quadripole $E \sim 1/r^4$

$$E_x = -\frac{\partial V}{\partial x} = \frac{kQx}{(x^2+a^2)^{3/2}}\hat{\imath}$$ (see page 688)

Disk
$$V = \int \frac{k\,dq}{r} = \int_{a=0}^{a=R} \frac{k}{(x^2+a^2)^{1/2}} \cdot 2\pi a \sigma\, da \qquad \sigma = \frac{Q}{\pi R^2}$$

$$= k\sigma\pi \int_0^R \frac{2a\,da}{(x^2+a^2)^{1/2}} = 2\pi k\sigma\left(\sqrt{x^2+R^2} - x\right)$$

$$V = 2\pi k\sigma |x| \left(\left[1+\frac{R^2}{x^2}\right] - 1\right) \to 2\pi k\sigma |x| \left(\frac{1}{2}\frac{R^2}{x^2}\right)$$

$$V = \frac{kQ}{|x|}$$

Infinite Plane = Disk to infinite R

Disk $\quad E_x = \underbrace{2\pi k\sigma}_{\sigma/2\varepsilon_0} \left(\frac{x}{\sqrt{x^2}} - \frac{x}{\sqrt{x^2+R^2}} \right) = \frac{\sigma}{2\varepsilon_0}\left(\frac{x}{|x|} - \frac{x}{\sqrt{x^2+R^2}} \right)$

infinite plane: $\quad R >> x \quad E_x = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{R}\right) \rightarrow \sigma/2\varepsilon_0$ getting real close

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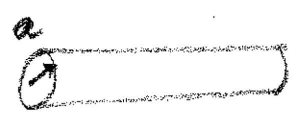
$x >> R$

$$E_x = \sigma/2\varepsilon_0 \left(1 - \frac{1}{(1 + R^2/x^2)} \right) = \frac{\sigma}{2\varepsilon_0}\left(1 - 1 + \frac{R^2}{2x^2} \right)$$

$$= \frac{\sigma}{2\varepsilon_0} \cdot \frac{R^2}{2x^2} = \frac{Q}{4\pi\varepsilon_0 x^2} = \frac{kQ}{x^2} \qquad Q = \pi R^2 \sigma$$

cylinder

$E = \frac{\lambda}{2\pi\varepsilon_0 r}$



$Q = 2\pi a l \sigma$

$\lambda = Q/l = 2\pi a \sigma$

$E = \frac{2\pi a \sigma}{2\pi\varepsilon_0 a} \rightarrow \sigma/\varepsilon_0$

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sphere

$E = \frac{q}{4\pi\varepsilon_0 r^2} \qquad q = 4\pi a^2 \sigma$

$\bar{E}(r=a) = \frac{4\pi a^2 \sigma}{4\pi\varepsilon_0 a^2} = \sigma/\varepsilon_0$

Potential – Infinite Disk

$$V = 2\pi k\sigma |x| \left[\left(1 + R^2/x^2\right)^{1/2} - 1 \right]$$

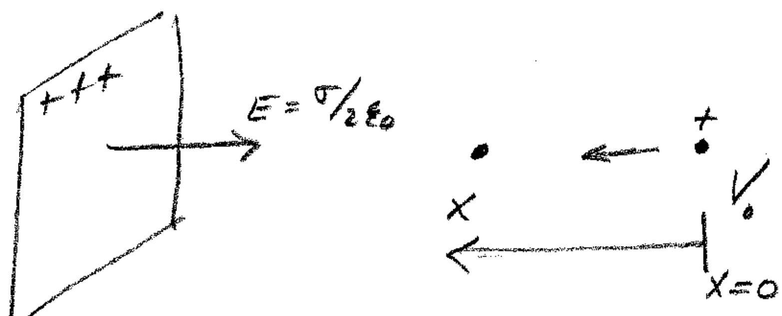
$x >> R$ $\quad V = 2\pi k\sigma |x| \left(\frac{R^2}{2x^2} \right) \rightarrow k\frac{Q}{x}$

$R >> x$ $\quad V = 2\pi k\sigma \left[(x^2 + R^2)^{1/2} - \sqrt{x^2} \right]$

$$E \rightarrow \sigma/2\varepsilon_0 = -\partial V/\partial x$$

$$V = \frac{\sigma}{2\varepsilon_0} x + V_0$$

at some reference



$$V = V_0 + \frac{\sigma}{2\varepsilon_0} x$$