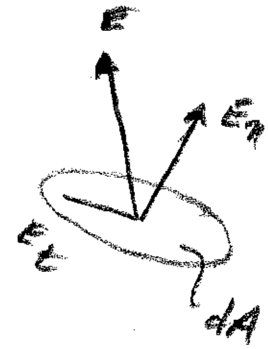
Lecture 24

Wed 03/21/07

Gauss' Law



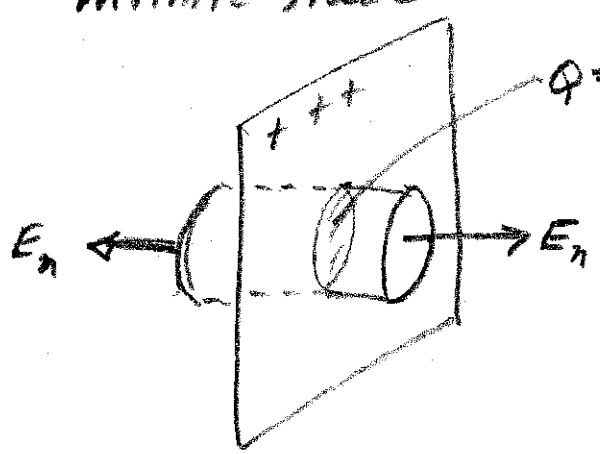
Flux $= \phi = E \cdot A = \int E \cdot \hat{n}\, dA$

$$\boxed{\phi = \int \bar{E} \cdot d\bar{A} = 4\pi k Q_{incl} = \frac{Q_{encl}}{\varepsilon_0}}$$

motivation $E_n = \frac{kQ}{R^2}$

$$\phi = \frac{kQA}{R^2} = kQ \cdot 4\pi = \frac{Q_{encl}}{\varepsilon_0}$$

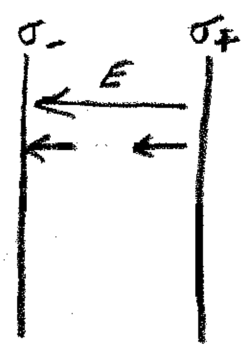
infinite sheet



$Q = \sigma A = \sigma \pi a^2$

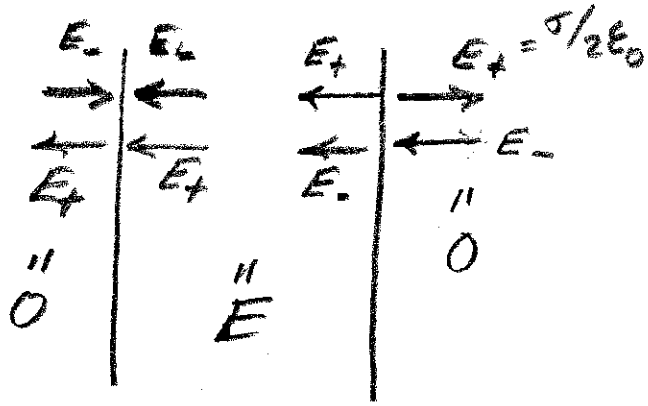
$$2E_n A = \frac{\sigma A}{\varepsilon_0} \qquad E_n = \frac{\sigma}{2\varepsilon_0}$$

parallel plate capacitor



$E = \frac{\sigma_+}{2\varepsilon_0} + \frac{\sigma_-}{2\varepsilon_0} = \sigma/\varepsilon_0$ inside

$\int E \cdot dA = 0 \Rightarrow \vec{E} = 0$ outside



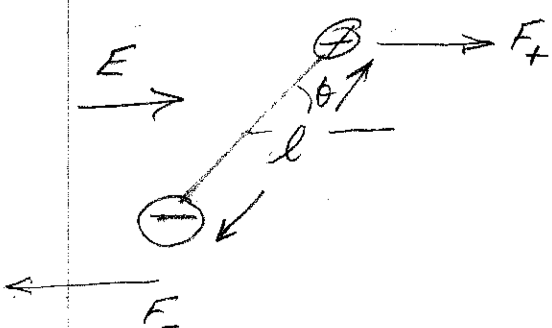
$F = qE$

$F = \sigma E_{dis} = \sigma\left(\frac{\sigma}{2\varepsilon_0}\right)$

$= \frac{\sigma^2}{2\varepsilon_0}$ don't get fooled

charges can't pull themselves up by their own bootstraps.

Electric Dipole



$F_+ = qE$ $\quad \Sigma F = 0$

$F_- = -qE$

$\tau = l\sin\theta F = \sin\theta\, q l E$

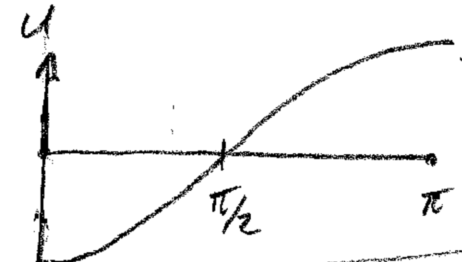
$\tau = q l E \sin\theta = p E \sin\theta$

$\tau = \vec{p} \times \vec{E}$ RH-rule

if $\vec{p}$ defined as $\ominus \longrightarrow \oplus$

Potential $\quad dW = \tau\, d\theta = -dU = -pE\sin\theta$

define $U = 0$ at $\theta = 90°$ $\qquad U = -pE\cos\theta = -\vec{p}\cdot\vec{E}$

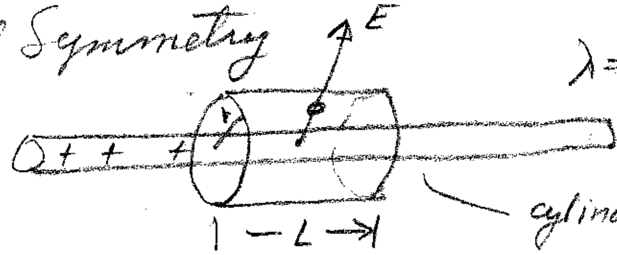


$U = qV - p\cdot E + Q_{ij}\frac{\partial E}{\partial x_j} + \cdots$

Cylindrical Symmetry

$\lambda = \frac{dQ}{dl}$

wire

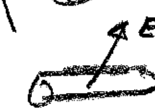
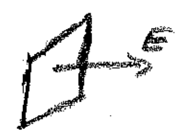


cylinder of radius "r"

outside

$$\int E \cdot dA = \frac{q}{\varepsilon_0} = E\, 2\pi r \cancel{L} = \frac{1}{\varepsilon_0}(\lambda \cancel{L})$$

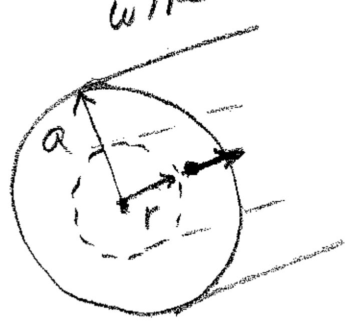
$$E = \frac{\lambda}{2\pi\varepsilon_0} \cdot \frac{1}{r}$$ falls as $1/r$

quad	⊕ ⊖ / ⊖ ⊕	$= 1/r^4$
dipole	⊕ ⊖	$1/r^3$
mono	⊕	$1/r^2$
line		$1/r$
		const $= \sigma/2\varepsilon_0$

Inside

inside non-conducting wire

what if conducting



$Q = \lambda L$

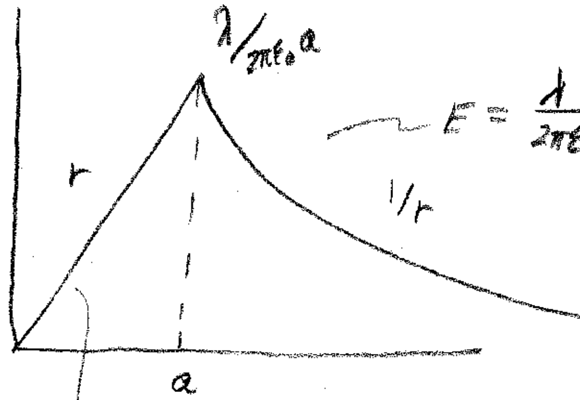
uniform density

$Q = L\pi a^2 \rho$

$\lambda = \frac{q}{L} = \pi r^2 \rho$

$$E \cdot 2\pi r \cancel{L} = \frac{1}{\varepsilon_0}(\pi r^2 \rho \cancel{L})$$

$$E = \left(\frac{\rho}{2\varepsilon_0}\right) \cdot r$$ linear in r



$E = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r}$

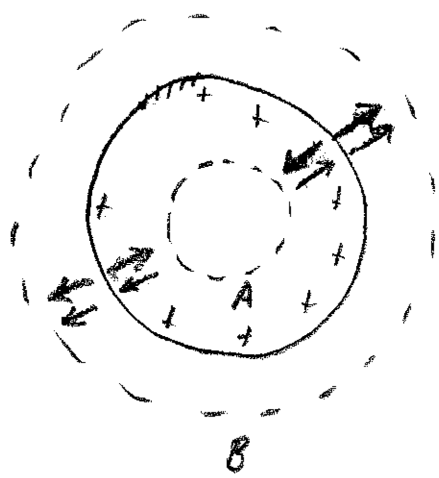
$$E_0(a) = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{a}$$

$$E_{in} = \left(\frac{\rho}{2\varepsilon_0}\right) r \rightarrow E_{in}(a) = \frac{\rho}{2\varepsilon_0} a$$

$$\left[\begin{array}{l} \rho = \frac{Q}{L\pi a^2} \\ \quad = \frac{\lambda}{\pi a^2} \end{array}\right] \quad E_{in} = \frac{1}{2\varepsilon_0}\frac{\lambda}{\pi a^2}\cdot a \Rightarrow \frac{\lambda}{2\pi\varepsilon_0 a}$$

Spherical Shell - Solid sphere!

shell



inside = A

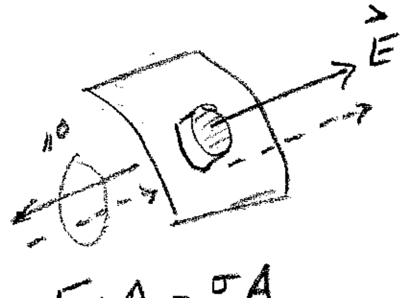
$$E \cdot 4\pi r_A^2 = \frac{Q}{\varepsilon_0} = 0$$

$$\vec{E}_a = 0$$

outside = B

$$E \cdot 4\pi r_B^2 = \frac{Q}{\varepsilon_0} \rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_B^2}$$

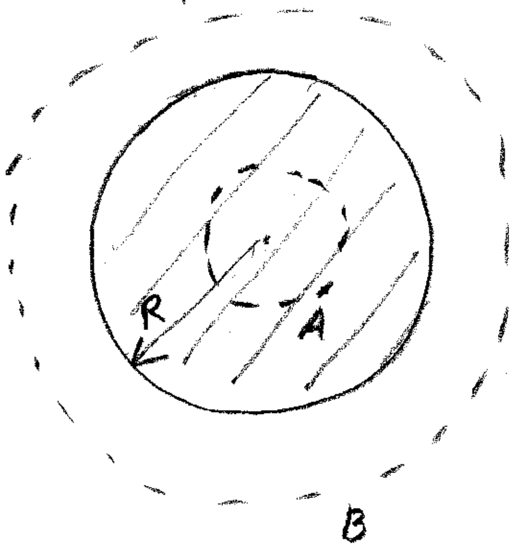
gauss



$$E = \frac{1}{4\pi\varepsilon_0}\frac{4\pi r_B^2 \sigma}{r_B^2} = \sigma/\varepsilon_0$$

$$E \cdot A = \frac{\sigma}{\varepsilon_0} A \qquad E = \sigma/\varepsilon_0$$

Solid Sphere



A = inside

$$E \cdot 4\pi r_A^2 = \frac{1}{\varepsilon_0} Q_{inclosed}$$

$$= \frac{1}{\varepsilon_0}\left\{ \rho \frac{4}{3}\pi r_A^3 \right\}$$

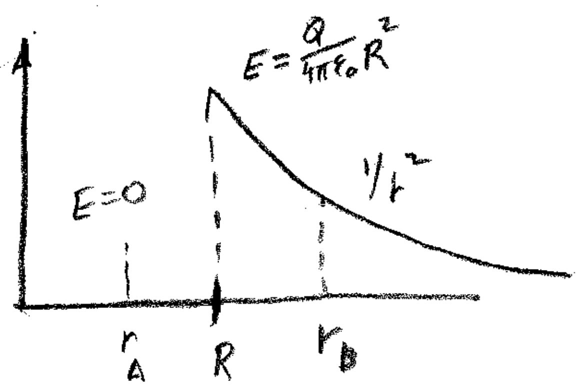
$$E = \frac{\cancel{4\pi}\rho r_A^3}{3 \cdot \cancel{4\pi}\varepsilon_0 r_A^2} = \frac{\rho}{3\varepsilon_0} r_A$$

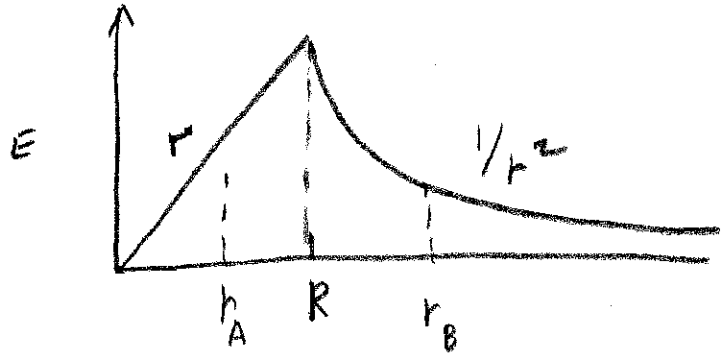
$$Q_{tot} = \frac{4}{3}\rho\pi R^3 \qquad E = \frac{Q}{4\pi\varepsilon_0 R^3} \cdot r_A \longrightarrow \frac{Q}{4\pi\varepsilon_0 R^2}$$

at surface

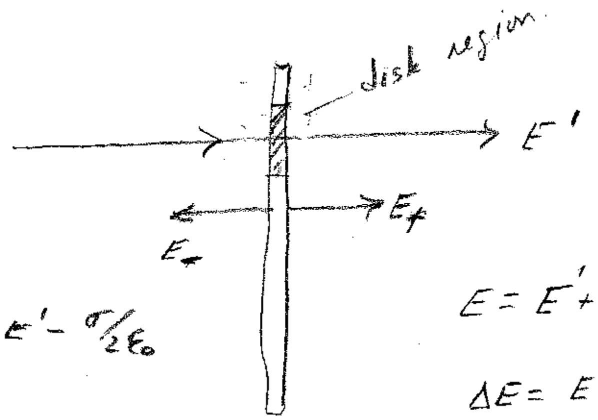
B = outside

$$E\left(4\pi r_B^2\right) = \frac{Q}{\varepsilon_0} \qquad E_B = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_B^2}$$





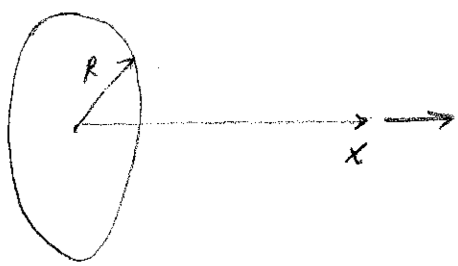
Discontinuities of $\vec{E}$



$E = E' - \sigma/2\varepsilon_0$

$$E = E' + E_+ = E' + \sigma/\varepsilon_0$$

$$\Delta E = E' + \sigma/2\varepsilon_0 - (E' - \sigma/2\varepsilon_0) = \underline{\sigma/\varepsilon_0} \quad \text{discontinuity}$$

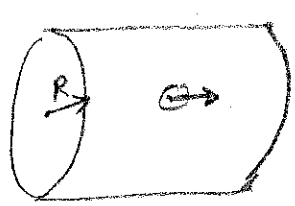


$$E_x = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{1 + \frac{R^2}{x^2}}}\right)$$

as $x \to 0$

$$E_x \to \frac{\sigma}{2\varepsilon_0}$$

cylinder



$$E = \frac{\lambda}{2\pi\varepsilon_0 R} = \frac{2\pi R\sigma}{2\pi\varepsilon_0 R} \to \sigma/\varepsilon_0$$

<u>Conductors</u> – charges reside on surfaces. (<u>statics</u>)