Lecture ~~20~~ 23

Monday

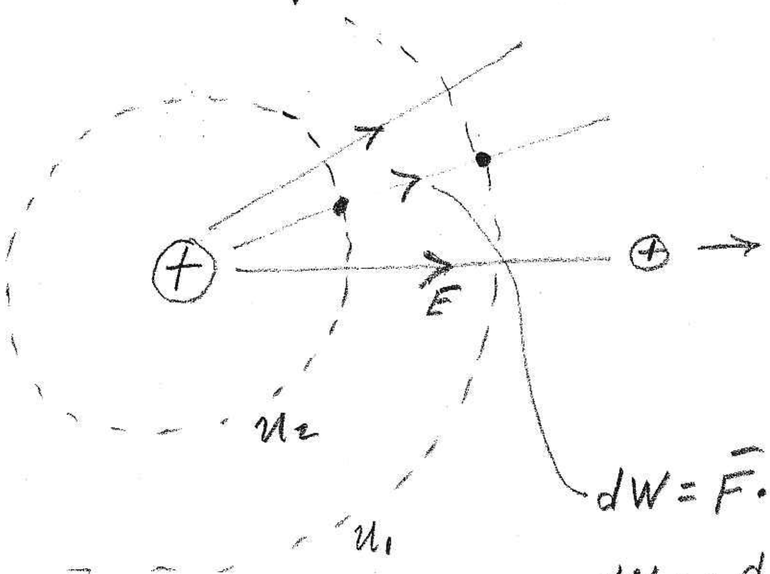
Chap 22

$$F = k\frac{qq'}{r^2}\hat{r} = \underbrace{\frac{1}{4\pi\varepsilon_0}}_{9\times10^9}\frac{qq'}{r^2}\hat{r}$$

$\varepsilon_0 = 8.85\times10^{-12}$ farads/m
permittivity of free space

$$E = \frac{F}{q'} = \frac{kq}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$e = 1.6\times10^{-19}$ coulombs.
Amp = coul/sec



$dW = \vec{F}\cdot d\vec{r}$

$dU = -\frac{dW}{q'} = -\vec{E}\cdot d\vec{r}$

$F = -kx$
mechanical analogy.

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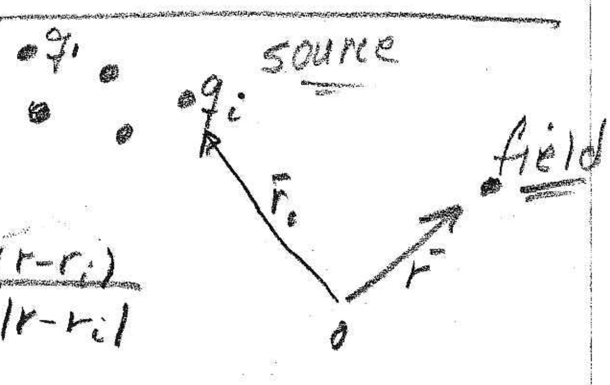
charge distributions



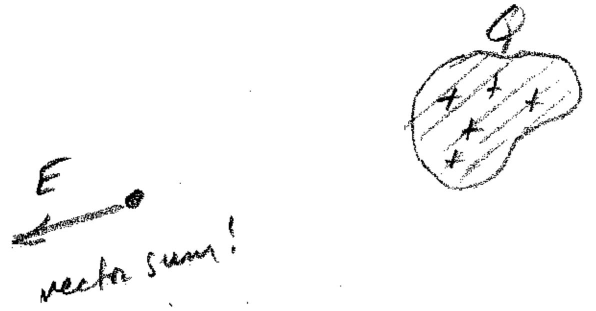
$$E = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{|\bar{r}-\bar{r}_i|^2}\frac{(r-r_i)}{|r-r_i|}$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i^2}\hat{r}_i$$

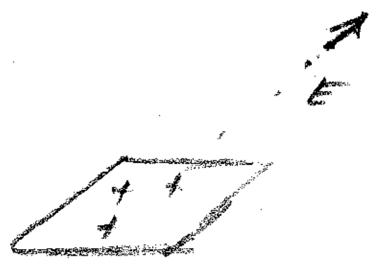
$$\sum q_i \rightarrow Q = \sum \rho_i(r)\,\Delta V \qquad \rho_i = \frac{q_i}{\Delta V} = \frac{dQ}{dV}$$

$$E = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r)\,\hat{r}}{r^2}\,dV$$

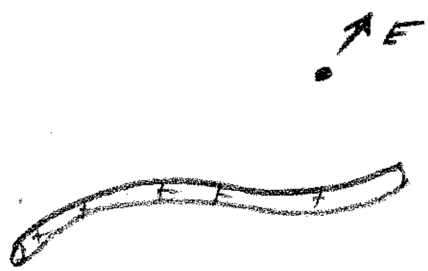
volume integral.



<u>volume</u> $\rho = \frac{dQ}{dV}$

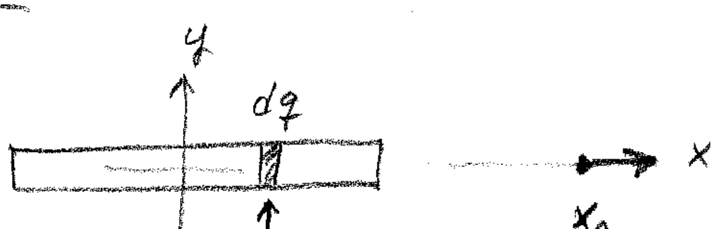


<u>surface</u> $\sigma = \frac{dQ}{dA}$



line $\lambda = \frac{dQ}{dL}$

<u>Example</u>



$$dE_x\,\hat{i} = \frac{k\,dq}{(x-x_p)^2} = \frac{k\lambda\,dx}{(x-x_p)^2}$$

$$E_x = \int_{-L/2}^{L/2} dE_x = \frac{k\lambda L\,\hat{i}}{(x_p)^2-(L/2)^2} = \frac{kQ}{(x_p)^2-(L/2)^2}$$

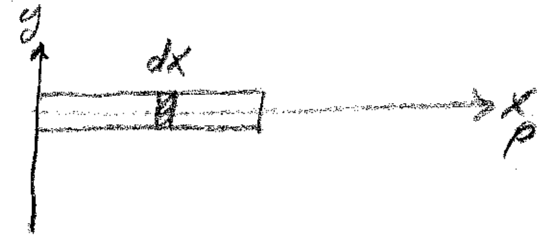
suppose $X_p >> L/2$

$$E_x = \frac{k\lambda L}{(X_p^2)\left[1-\left(\frac{L}{2X_p}\right)^2\right]} = \frac{k\lambda L}{(X_p)^2}\left\{1+\frac{L^2}{4X_p^2}\right\}$$

using $(1 \pm x)^m = 1 \pm mx$
binomial expansion

$$= \frac{kQ}{X_p^2} + \frac{k\lambda L^3}{4X_p^4}$$

monopole   quadrupole



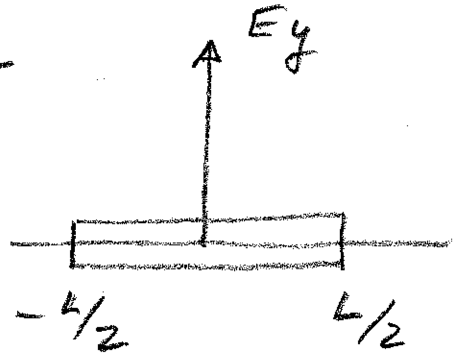
$$E_x = \int_0^L \frac{k\lambda\, dx}{(x-x_p)^2} = \frac{k\lambda}{X_p - L} - \frac{k\lambda}{X_p}$$

$$= \frac{k\lambda X_p - k\lambda(X_p - L)}{X_p(X_p - L)} = \frac{k\lambda L}{X_p^2\left(1-\frac{L}{X_p}\right)}$$

$$= \frac{k\lambda L}{X_p^2}\left(1+\frac{L}{X_p}\right) = \frac{kQ}{X_p^2} + \frac{kQL}{X_p^3}$$

dipole

show



show $L/2 \to \infty$ $\quad E_y = \dfrac{2k\lambda}{R}$

$Q \to \infty$

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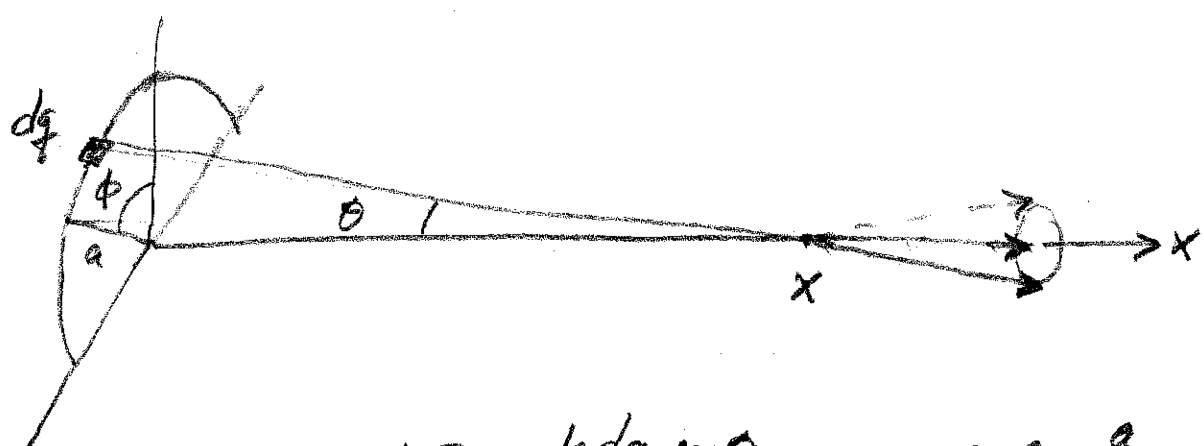
ring of charge



$$dE_x = \frac{k\,dq}{r^2}\cos\theta \qquad \cos\theta = \frac{a}{r}$$

$Q = 2\pi a\lambda$

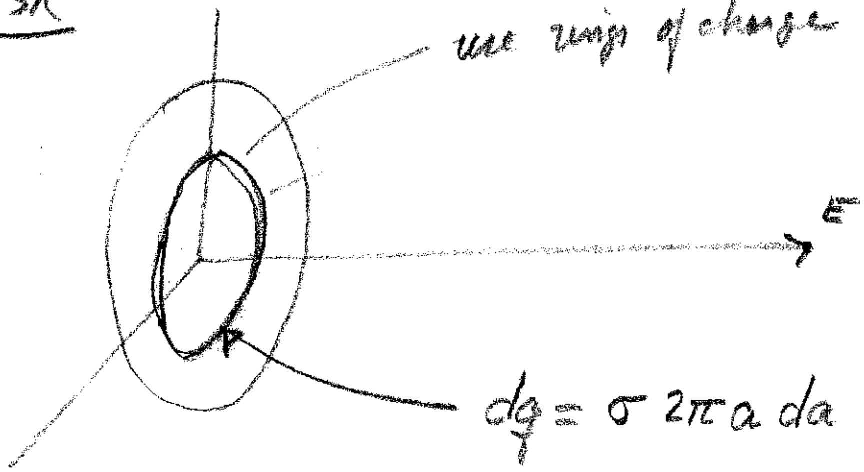
$dq = 2\pi\lambda a\,d\phi$

$\int dq = Q$

$$dE_x = \frac{k\,dq\,a}{(x^2+a^2)^{3/2}}$$

$$E_x = \int \frac{k\,x\,dq}{(x^2+a^2)^{3/2}} = \frac{kQx}{(x^2+a^2)^{3/2}}$$

<u>disk</u>



$$dE = \frac{k(2\pi a\,da)\,x}{(x^2+a^2)^{3/2}} = \frac{dE_x}{da}\,da$$

$$E_x = \int_{a=0}^{a=R} \frac{dE_x}{da}\,da = 2\pi k\sigma\left(1 - \frac{1}{\sqrt{1+R^2/x^2}}\right)$$

<u>infinite sheet</u> let $R \to \infty$

$$\frac{1}{\sqrt{1+R^2/x^2}} = \left(1+\frac{R^2}{x^2}\right)^{-1/2} \approx 1 - \frac{1}{2}\frac{R^2}{x^2}$$

$$E_x = 2\pi k\sigma\left(1 - 1 + \frac{R^2}{2x^2}\right) = \frac{\pi k\sigma R^2}{x^2}$$

$$E_x = \frac{kQ}{x^2} \quad \rightarrow \quad 2\pi k\sigma = \frac{\sigma}{2\varepsilon_0}$$

infinite sheet