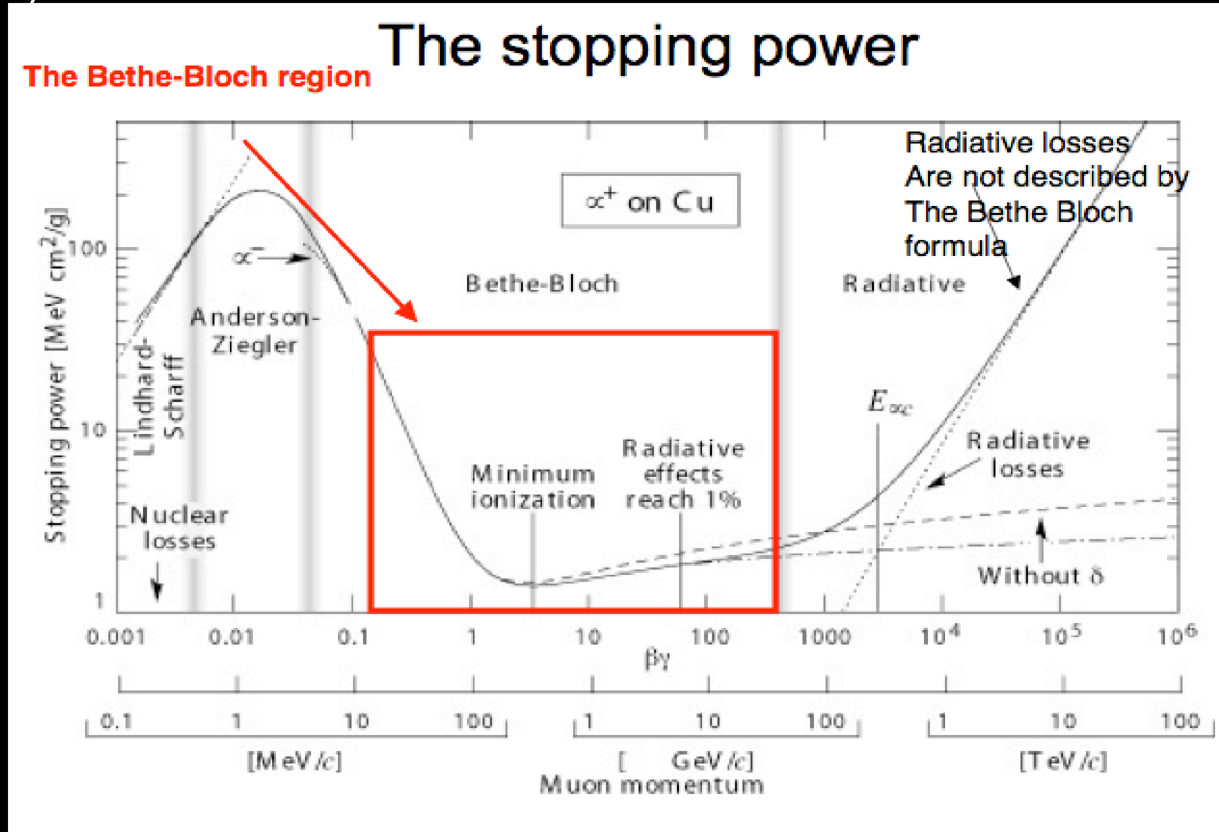
Bethe-Block

Stopping power of positive muons in copper vs $\beta\gamma$ = p/Mc. The slight dependence on M at highest energies through T_{max} can be used for PID but typically dE/dx depend only on β (given a particle and medium)



At low β $-dE/dx \propto 1/\beta^2$ decreases rapidly as β increases. At relativistic velocities $\beta \approx 1$ and reaches a min at $\beta\gamma \approx 3$ (a particle at the energy loss min is called mip). Beyond the min the energy loss increases logarithmically (due to the increase of T_{max} and b_{max}).

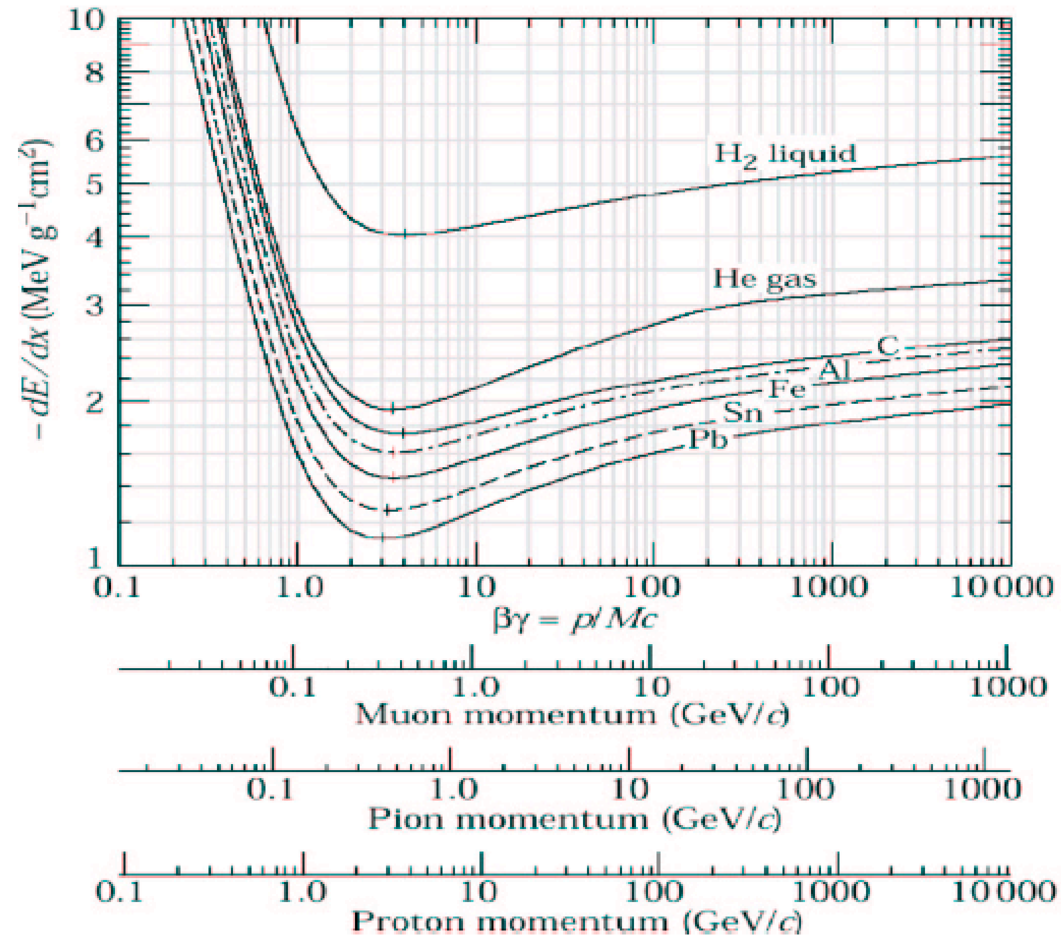
However as the range of distant collisions extends, the atoms close to the path of the particle will produce a polarization which results in reducing the electric field strength acting on electrons at large distances Density effect: $\delta/2$

The relativistic rise depends on $\ln(\beta\gamma)$ but in the ultrarelativistic region only on $\ln\gamma$ hence on the particle mass (used for PID)

Bethe-Block

For a given particle (z) and target (I,N,Z,A), the energy loss depends only on the velocity of the particle!

Energy losses: dE/dx(MeV/g•cm^{-2})



Important features

- The Bethe-Bloch formula works well from:
  $0.1 < \gamma\beta < 100$
- The minimum ionizing energy loss is about
  1 - 4 MeV/g cm^{-2}
  ranging from H to Pb (as target material!).
  This is an important calibration tool. One speaks of "minimum ionizing particles". They can be used to calibrate detectors, e.g. cosmic ray muons in scintillators.
- The energy loss rises slowly - the "logarithmic rise". This is caused by the increase of the transverse field of the relativistic particle.
- Once the particles energy drops below that value it tends to stop quickly. E. g., use low energy pions to deposit most of the energy at a well defined target point to hit a brain tumor, without causing much radiation damage before the target.

Most relativistic particles have energy loss rates close to the minimum (mip = minimun ionizing particles)~ 2 MeV/g/cm^2

δ rays and restricted energy loss

The kinetic energy of the particle is transferred to electrons (δ-rays) emitted in directions close to the incoming particle one ($\cos\theta \propto 1/T_{max}$). Therefore the energy of the particle can result in secondary ionization processes with additional δ-rays traveling far away from the emission point. Often a restricted energy loss is used to estimate the deposited energy in a detector using instead of T_{max} the effective detectable max transferred energy (= effective average max δ-rays energy that can be absorbed inside the device).

$$-\left.\frac{dE}{dx}\right|_{T<T_{\rm cut}} = K z^2 \frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2T_{\rm cut}}{I^2} - \frac{\beta^2}{2}\left(1+\frac{T_{\rm cut}}{T_{\rm max}}\right) - \frac{\delta}{2}\right] .$$

Fluctuations in energy losses:

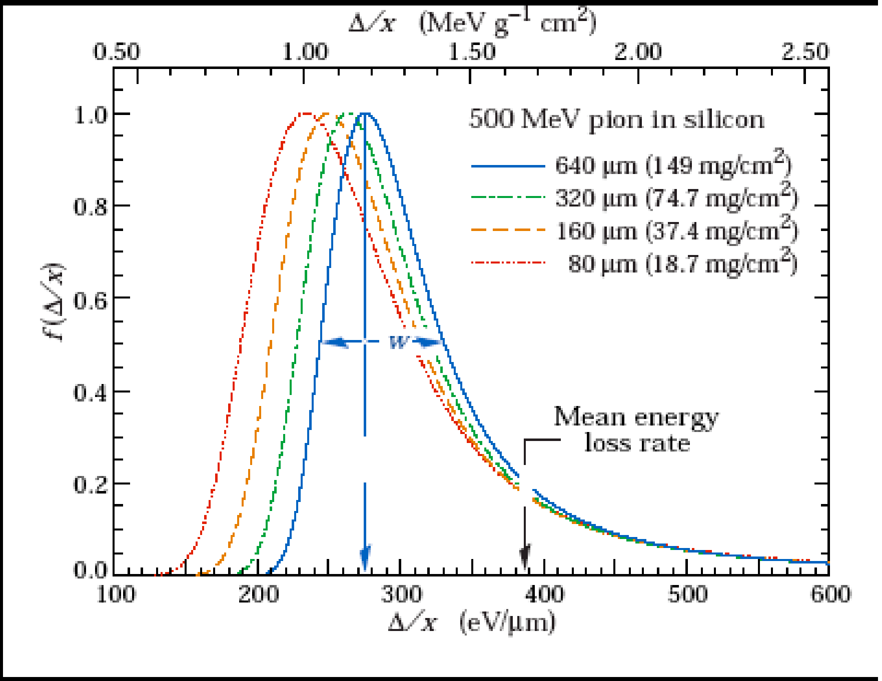
Energy losses of massive charged particles are a statistical phenomenon (collisions are a series of independent events) and in each interaction different amounts of kinetic energy can be transferred to atomic electrons. The energy lost by a particle crossing a path x has an energy distribution called energy struggling function (not for electrons for which the collision process is not dominant) which is the solution of transport equation:

$$f(\Delta; \beta\gamma, x)$$

The pdf describing the distribution of energy loss Δ in an absorber thickness x is called Landau distribution

The most probable energy loss $\Delta_p \propto x(a+\ln x)$ and the ratio of the full width at half maximum (w = FWHM) w/ $\Delta_p \propto 1/x$

For very thick absorbers where the energy loss exceeds 1/2 Einitial f(Δ) approximates a Gaussian



Range

Range of charged particles in liquid H_2, He gas, Carbon, Iron, lead

A proton of 10 GeV

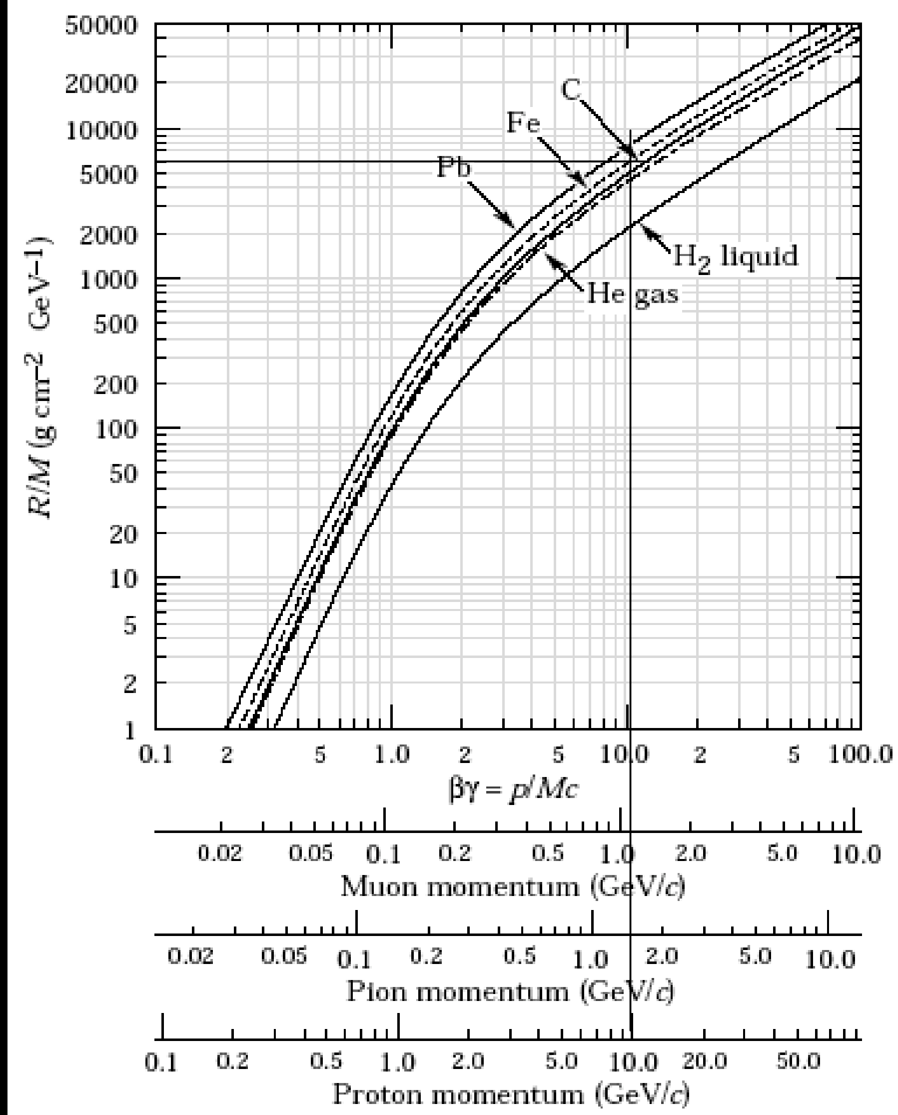
M ~1 GeV will cross R = 6000 g/cm^2 in Fe(true if interaction length > R)

$$R_{\langle \Delta E \rangle}(E_0) = \int_0^{E_0} \frac{dE_\mu}{\langle dE(E_\mu)/dX \rangle}$$

At high energy

-dE/dx depends on $\beta \Rightarrow$ R depends on $\beta E \Rightarrow \beta E = pc \Rightarrow$ R/M depends on pc/M = $\beta\gamma$

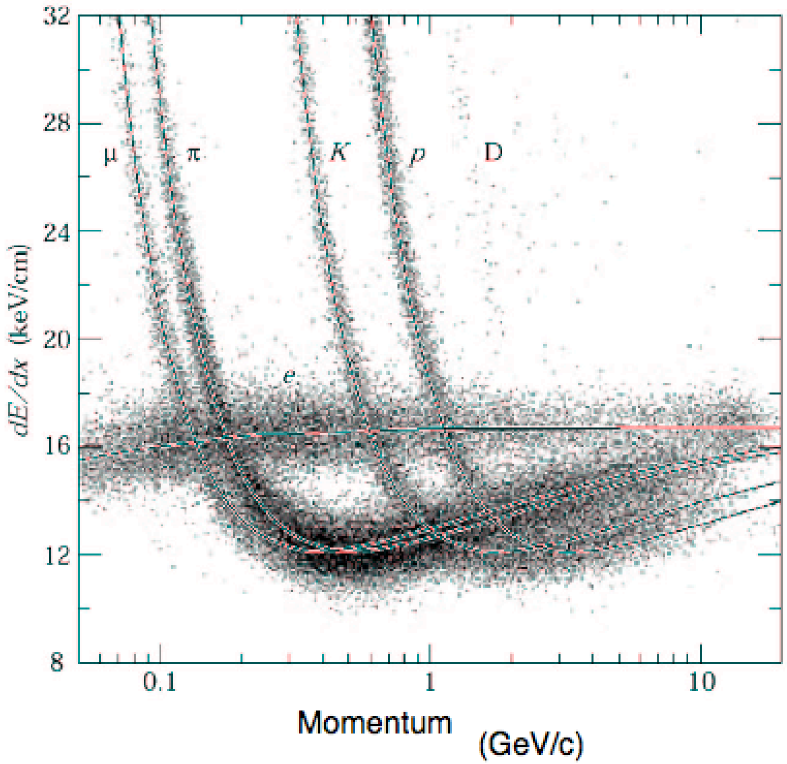
Due to the statistical nature of energy loss, mono-energetic particles do not travel the same paths in absorbers (range straggling that follows a Gaussian distribution)



PID using dE/dx

The low momentum region where $-dE/dx \propto 1/\beta^2$ and the relativistic rise depend on m so can be used for PID. While in 1 thin layer of absorber the mean of the energy loss distribution is affected by the long tail of the Landau asymmetric distribution, many consecutive layers can be used to make a precise measurement of energy loss that allows (for known momentum) to determine particle masses. For a fixed layer thickness L/N of N chambers the resolution improves as 1/sqrt(NL)

Plot dE/dx versus momentum of particle



Radiation Energy Losses of Electrons and Positrons

At low energies electrons and positrons lose energy by ionization and scattering processes (Møller, Bhabba, e+ annihilation). Ionization energy losses rise logaritmically with energy, while radiative losses rise almost linearly so high energy electrons ($E_{critical}$ ~550/Z in MeV for Z>13) and positrons loose energy by radiative emissions (bremsstrahlung when traversing matter and synchrotron radiation when occurring in circular acceleration) and e^+e^- pair production. Radiation emission is connected to acceleration or deceleration of charged particles.The emitted radiation per unit time depends on the velocity variation:

$$\frac{dE}{dt} = \frac{2e^2}{3c^3}\left|\frac{d\mathbf{v}}{dt}\right|^2$$

For bremsstrahlung by a particle of charge z and mass m in an absorber of atomic number Z the acceleration depends on zZe^2/m_p and hence it is much less probable for massive particles than electrons

$$\left(\frac{dE}{dt}\right)_B \propto \frac{z^2Z^2}{m^2}$$

The characteristic amount of matter traversed while bremsstrahlung is called radiation length X_0 (in g/cm²): the layer thickness that reduces the mean energy by a factor of e:

$$\frac{dE}{E} = -\frac{dx}{X_0} \Rightarrow \langle E \rangle = E_0 e^{-X/X_0}$$

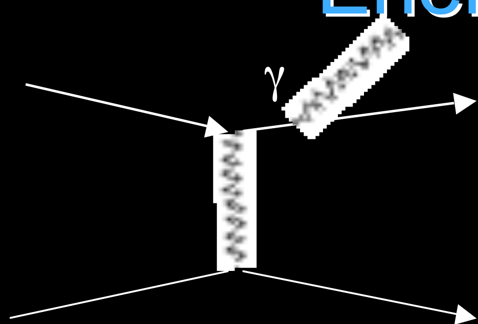
and pair productions occur with a characteristic length of L_p~9/7X_0

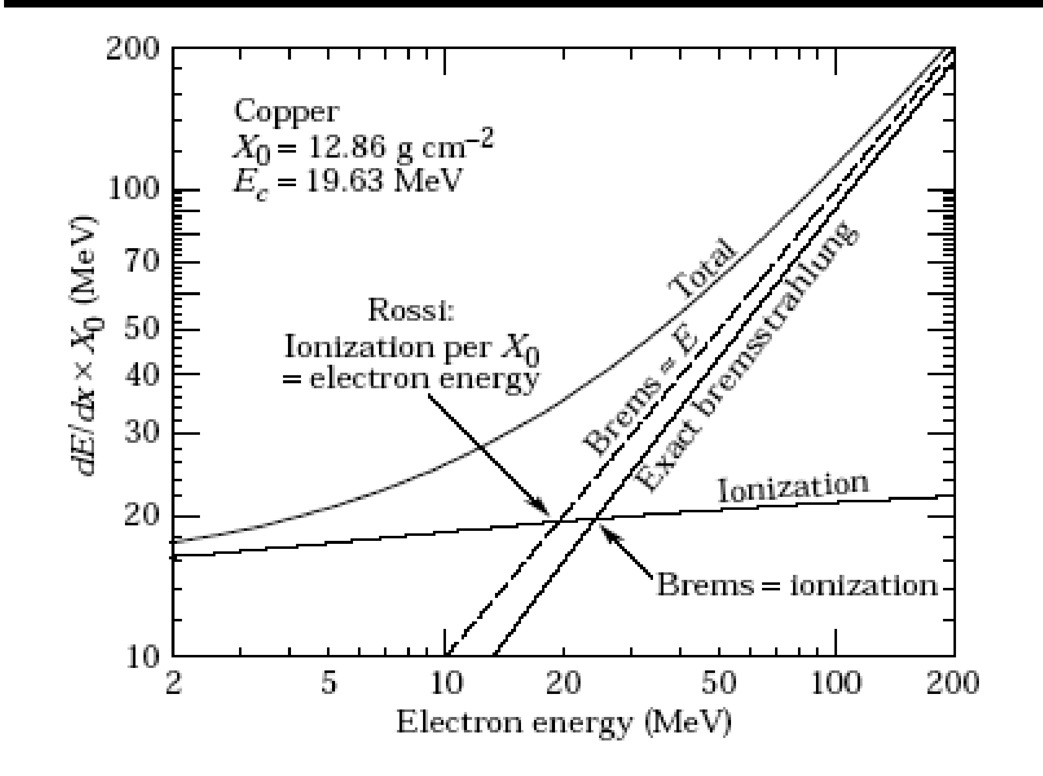
$$\sigma_{bremss}(E > m_e) \approx \alpha^3 z^2 4 r_e^2 \left(\ln\frac{183}{Z^{1/3}} + \frac{1}{8}\right)$$

$$X_0(g/cm^2) = \frac{716A}{Z(Z+1.3)\left(\ln\frac{183}{Z^{1/3}} + \frac{1}{8}\right)} = \frac{A}{N_A\sigma_{bremss}}$$

Z(Z+1.3) accounts for interactions between high energy electron and those bound to atom

Energy losses of electrons





The process can happen only in the field of an atom or of atomic electrons
to conserve energy-momentum

ATMOSPHERE 38 g/cm^2 => 280 m => atmosphere is about 25 X_0 at the vertical

LEAD 6.37 g/cm^2/ 11.35 g/cm^3 = 0.56 cm

Atomic and Nuclear Properties of materials

Material	Z	A	$\langle Z/A \rangle$	Nuclear [a] collision length λ_T {g/cm^2}	Nuclear [a] interaction length λ_I {g/cm^2}	$dE/dx\|_{min}$ [b] $\left\{\frac{MeV}{g/cm^2}\right\}$	Radiation length [c] X_0 {g/cm^2}	Radiation length X_0 {cm}	Density {g/cm^3} ({g/ℓ} for gas)	Liquid boiling point at 1 atm(K)	Refractive index n (($n-1$)×10^6 for gas)
H_2 gas	1	1.00794	0.99212	43.3	50.8	(4.103)	61.28 [d]	(731000)	(0.0838)[0.0899]		[139.2]
H_2 liquid	1	1.00794	0.99212	43.3	50.8	4.034	61.28 [d]	866	0.0708	20.39	1.112
D_2	1	2.0140	0.49652	45.7	54.7	(2.052)	122.4	724	0.169[0.179]	23.65	1.128 [138]
He	2	4.002602	0.49968	49.9	65.1	(1.937)	94.32	756	0.1249[0.1786]	4.224	1.024 [34.9]
Li	3	6.941	0.43221	54.6	73.4	1.639	82.76	155	0.534		—
Be	4	9.012182	0.44384	55.8	75.2	1.594	65.19	35.28	1.848		—
C	6	12.011	0.49954	60.2	86.3	1.745	42.70	18.8	2.265 [e]		—
N_2	7	14.00674	0.49976	61.4	87.8	(1.825)	37.99	47.1	0.8073[1.250]	77.36	1.205 [298]
O_2	8	15.9994	0.50002	63.2	91.0	(1.801)	34.24	30.0	1.141[1.428]	90.18	1.22 [296]
F_2	9	18.9984032	0.47372	65.5	95.3	(1.675)	32.93	21.85	1.507[1.696]	85.24	[195]
Ne	10	20.1797	0.49555	66.1	96.6	(1.724)	28.94	24.0	1.204[0.9005]	27.09	1.092 [67.1]
Al	13	26.981539	0.48181	70.6	106.4	1.615	24.01	8.9	2.70		—
Si	14	28.0855	0.49848	70.6	106.0	1.664	21.82	9.36	2.33		3.95
Ar	18	39.948	0.45059	76.4	117.2	(1.519)	19.55	14.0	1.396[1.782]	87.28	1.233 [283]
Ti	22	47.867	0.45948	79.9	124.9	1.476	16.17	3.56	4.54		—
Fe	26	55.845	0.46556	82.8	131.9	1.451	13.84	1.76	7.87		—
Cu	29	63.546	0.45636	85.6	134.9	1.403	12.86	1.43	8.96		—
Ge	32	72.61	0.44071	88.3	140.5	1.371	12.25	2.30	5.323		—
Sn	50	118.710	0.42120	100.2	163	1.264	8.82	1.21	7.31		—
Xe	54	131.29	0.41130	102.8	169	(1.255)	8.48	2.87	2.953[5.858]	165.1	[701]
W	74	183.84	0.40250	110.3	185	1.145	6.76	0.35	19.3		—
Pt	78	195.08	0.39984	113.3	189.7	1.129	6.54	0.305	21.45		—
Pb	82	207.2	0.39575	116.2	194	1.123	6.37	0.56	11.35		—
U	92	238.0289	0.38651	117.0	199	1.082	6.00	≈0.32	≈18.95		—
Air, (20°C, 1 atm.), [STP]			0.49919	62.0	90.0	(1.815)	36.66	[30420]	(1.205)[1.2931]	78.8	(273) [293]
H_2O			0.55509	60.1	83.6	1.991	36.08	36.1	1.00	373.15	1.33

http://pdg.lbl.gov/2005/reviews/atomicrpp.pdf

Multiple Coulomb Scattering

When a charged particle passes in the neighborhood of a nucleus its trajectory is deflected. The Coulomb scattering distribution is well described in the Moliere Theory. For small scattering angles it is roughly Gaussian, but at larger angles It has larger tails than a Gaussian. Instead of considering the total deflection θ it is convenient to consider its projection on a plane. In the Gaussian approx (small deflection angles), the root mean square value (rms) projected on a plane is

$$\theta_0 = \theta^{rms}_{proj} = \sqrt{\langle \theta^2_{proj} \rangle} = \sqrt{\frac{1}{2}\langle \theta^2 \rangle} = \frac{1}{\sqrt{2}}\theta^{rms}$$

$$\theta_0 = \frac{13.6 \text{ MeV}}{\beta c p}\, z\, \sqrt{x/X_0}\Big[1 + 0.038 \ln(x/X_0)\Big] \; .$$

$$\theta^2_{\text{space}} \approx (\theta^2_{\text{plane},x} + \theta^2_{\text{plane},y}),$$

x,y orthogonal to direction of motion

The space and plane angular distributions are approximately given by

$$\frac{1}{\sqrt{2\pi}\,\theta_0} \exp\left(-\frac{\theta^2_{\text{plane}}}{2\theta_0^2}\right) d\theta_{\text{plane}} \; ,$$

$$\theta^2_{space} \approx 2\theta^2_{plane}$$

$$d\Omega \approx d\theta_{\text{plane},x}\, d\theta_{\text{plane},y} .$$

Product of 2 distributions on the plane

$$\frac{1}{2\pi\theta_0^2}\exp\left(-\frac{2\theta^2_{plane}}{2\theta_0^2}\right) d\theta^2_{plane}$$

→

$$\frac{1}{2\pi\,\theta_0^2} \exp\left(-\frac{\theta^2_{\text{space}}}{2\theta_0^2}\right) d\Omega$$

Muon energy losses

$$-dE/dx = a(E) + b(E)\,E$$

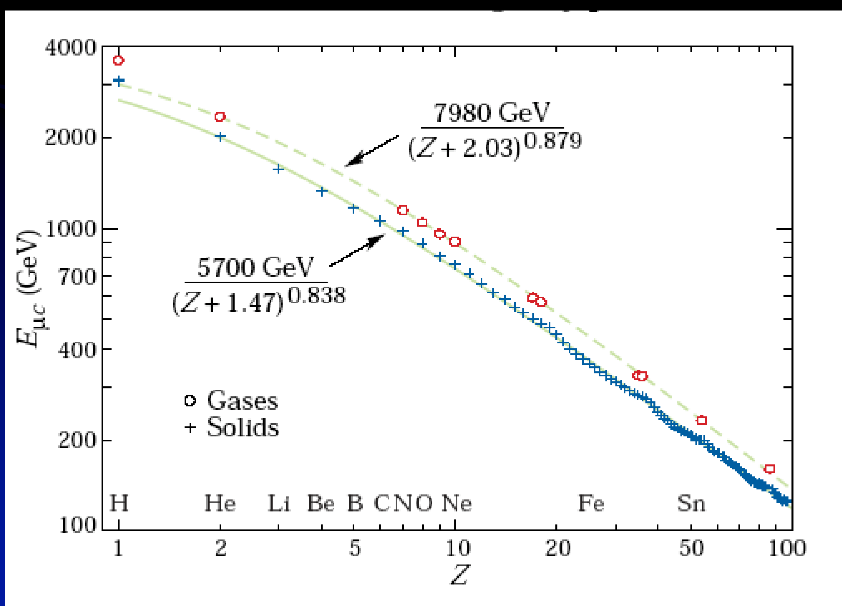
Ionization — Stochastic losses

~2 MeV/g/cm^2 (dominate > 1TeV)

$$R_\mu = \int_0^E \frac{dx}{dE} dE \approx \int_0^E \frac{1}{a+bE} dE = \frac{1}{b}\log(1+E/E_c)$$

$$E_c = a/b$$

critical energy



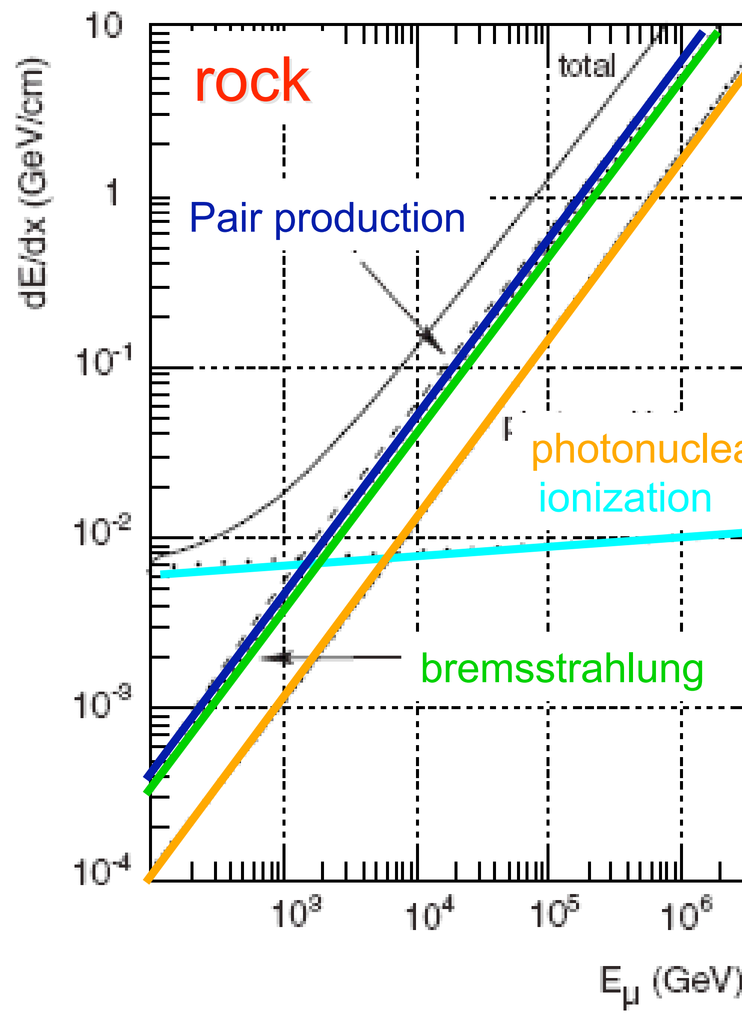


Table 23.2: Average muon range R and energy loss parameters calculated for standard rock [38]. Range is given in km-water-equivalent, or 10^5 g cm^{-2}.

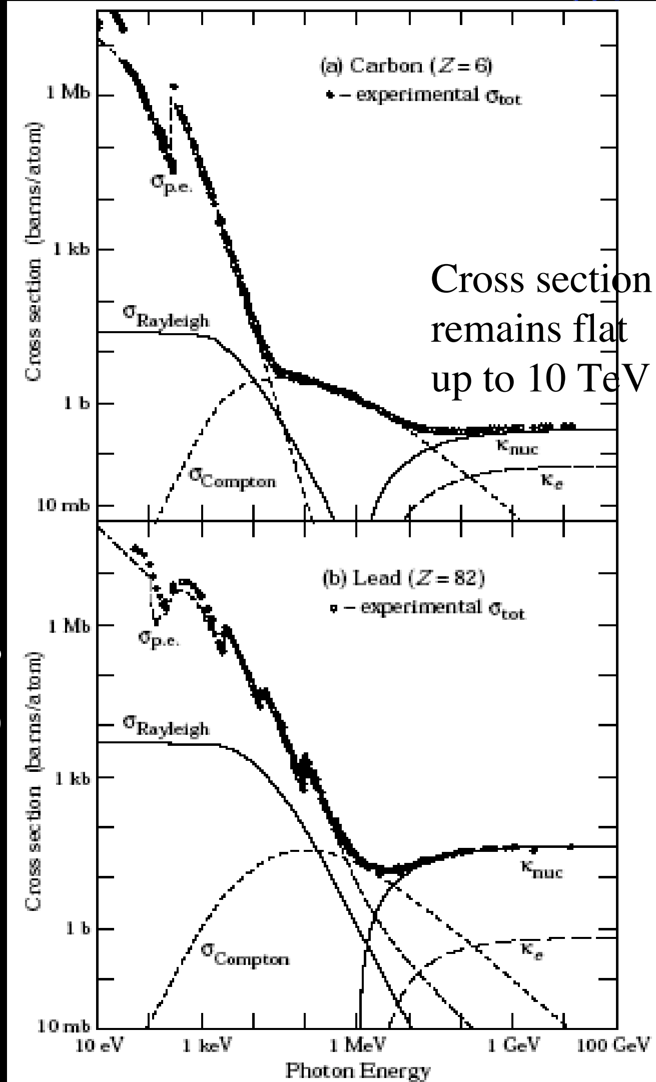
E_μ GeV	R km.w.e.	a MeV g^{-1} cm^2	b_{brems}	b_{pair}	b_{nucl}	$\sum b_i$
			——	10^{-6} g^{-1} cm^2		——
10	0.05	2.17	0.70	0.70	0.50	1.90
100	0.41	2.44	1.10	1.53	0.41	3.04
1000	2.45	2.68	1.44	2.07	0.41	3.92
10000	6.09	2.93	1.62	2.27	0.46	4.35

Interaction of Radiation with Matter

Beams of monochromatic photons of initial intensity I_0 are reduced exponentially in intensity $I = I_0 \exp(-\mu_{att}\, x)$ μ_{att} =attenuation coefficient = $\sigma N_A \rho/A$

The main processes contributing to the total cross-section are:

- Photoelectric effect: interactions with entire atomic electron cloud resulting in complete absorption of photon energy
- Thompson and Compton scattering on atomic electrons at γ energies >> electron binding energies and electrons are 'quasi-free'
- pair production: the photon incoming energy is high enough to allow e^+e^- creation in the Coulomb field of an electron or of a nucleus



$\sigma_{p.e.}$ = Atomic photoelectric effect (electron ejection, photon absorption)
$\sigma_{Rayleigh}$ = Coherent scattering (Rayleigh scattering—atom neither ionized nor excited)
$\sigma_{Compton}$ = Incoherent scattering (Compton scattering off an electron)
κ_{nuc} = Pair production, nuclear field
κ_e = Pair production, electron field

The photoelectric effect

1905 Einstein

The phenomenon is responsible of opacity in stellar interiors and atmospheres. At low photon energies $h\nu << m_e c^2$ it is the dominant process by which photons lose energy. If the energy of the incident photon is $h\nu$ it can eject electrons with binding energy $B_e \le h\nu$. The energy of an electron leaving the atom is $K_e = h\nu - B_e$. If the electron energy is lower than the binding energy of a shell an electron from that shell cannot be emitted. Therefore the absorption curve exhibits absorption edges whenever the incoming photon energy matches the ionization energy of K, L, M,…shells (that can have substructures except for K-shell) The photoelectric absorption probability is larger for more tightly bound electrons (K-shell) since free-electrons cannot absorb photons. The binding energy for K shell is $B_e(K) \approx R_y(Z-1)^2$ eV where R_y=13.61 eV Rydberg constant

The photoelectric effect

The K shell cross section ($h\nu << mc^2$) depends strongly on the medium!

If an atomic electron is emitted due to the photoelectric effect, a vacancy is created in the shell leaving the atom in an excited state. The atom can readjust itself to a more stable state via a radiative emission (X-ray photon) or 1 or more electrons (Auger effect)

$$\sigma_K = \sigma_{Th} 4\sqrt{2}\,\frac{Z^5}{137^4}\left(\frac{mc^2}{h\nu}\right)^{7/2}$$

$$\sigma_{Th} = (8/3)\pi r_e^2 = 6.6516\times10^{-25}\,cm^2$$

Compton Scattering

1923 Compton discovered that the wavelength of hard X-ray radiation increases when it is scattered from stationary electrons. This is another phenomenon in which radiation appears in its corpuscolar nature (duality wave-particle): scattering of photons on 'quasi-free'.

At lower energies Thompson scattering dominates in which the wavelength of photons is not changed. A classical description of the collision leads to a total cross section

$$\sigma_{Th} = \frac{8\pi}{3} r_e^2 = \frac{e^4}{6\pi\varepsilon_0^2 m_e^2 c^4} = 6.653 \times 10^{-29} m^2$$

The # of electrons/V $= N_e$ decreases exponentially with distance

$$-\frac{dN}{dx} = \sigma_T N_e N \Rightarrow N = N_0 \exp\left(-\int \sigma_T N_e dx\right)$$

optical depth of the medium due to Thompson scattering $\tau = \int \sigma_T N_e dx$

$$mc^2(\nu - \nu') = h\nu\nu'(1 - \cos\theta_\nu)$$

$$h\nu = h\nu' + \frac{h^2}{mc^2}\nu\nu'(1 - \cos\theta_\nu) = h\nu'\left[1 + \varepsilon(1 - \cos\theta_\nu)\right]$$

$\varepsilon = \frac{h\nu}{mc^2}$ reduced photon energy

Demonstrate the Compton shift formula $\Delta\lambda = \lambda' - \lambda = \lambda_e (1-\cos\theta_\nu)$ with $\lambda_e = \hbar/(mc)$ Compton wave length of electron

Compton Scattering

Hence the fraction of the incoming photon carried by the scattered photon is

$$\frac{h\nu'}{h\nu} = \frac{1}{[1+\varepsilon(1-\cos\theta_\nu)]} \Rightarrow \cos\theta_\nu = 1 - \frac{1}{\varepsilon}\left(\frac{h\nu'}{h\nu} - 1\right)$$

is the cosine of the photon scattering angle.

The photon is forward scattered when $h\nu \to h\nu'$ and for low energies ($\varepsilon << 1$) $h\nu \approx h\nu'$. Under these circumstances $K_e \approx 0$

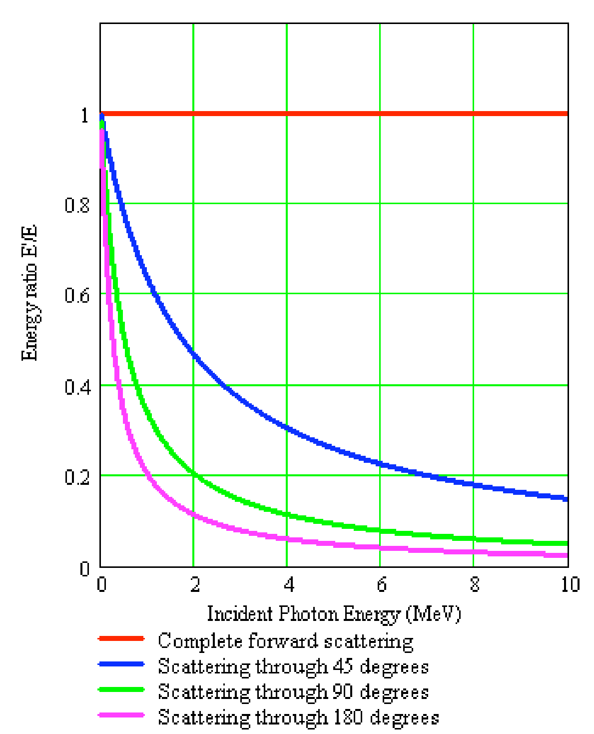
The electron kinetic energy is

$$K_e = h(\nu - \nu') = h\nu'[\varepsilon(1-\cos\theta_\nu)] = h\nu\frac{\varepsilon(1-\cos\theta_\nu)}{1+\varepsilon(1-\cos\theta_\nu)}$$

The maximum kinetic energy is for backward scattering of photons θ = 180deg

$$K_e = h\nu\frac{2\varepsilon}{1+2\varepsilon}$$

Fraction of the incident photon energy taken by the Scattered photon



The cross-section for Compton scattering

Klein-Nishina formula

$$\frac{d\sigma}{d\Omega} = 0.5r_e^2(P(E_\gamma,\theta) - P(E_\gamma,\theta)^2\sin^2(\theta) + P(E_\gamma,\theta)^3)$$

$$P(E_\gamma,\theta) = \frac{1}{1 + \frac{E_\gamma}{m_e c^2}(1-\cos\theta)}$$

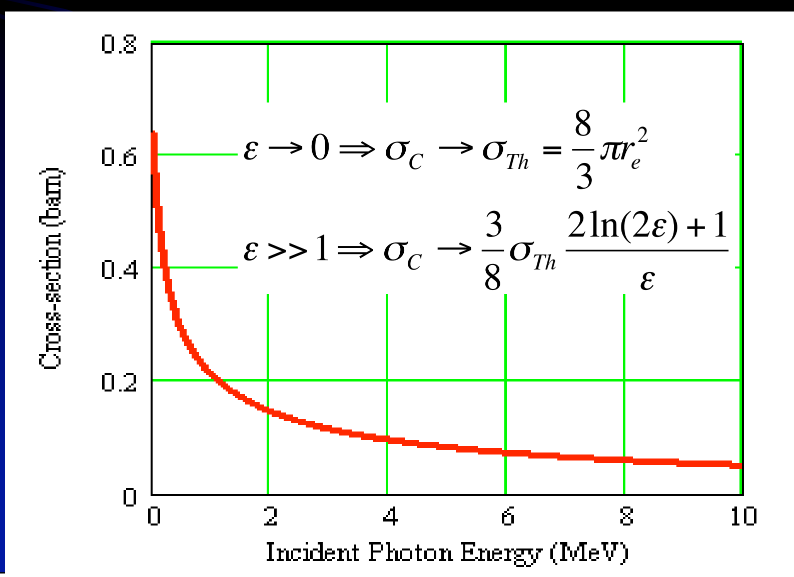
For an atom with Z electrons $\sigma = Z\,\sigma_{K\text{-}N}$

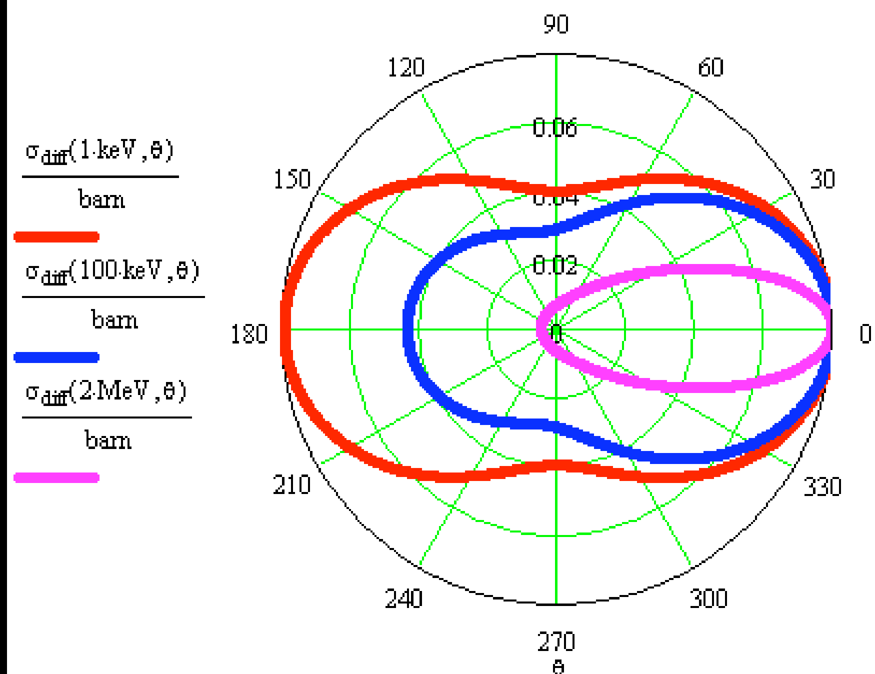
For low energies $\varepsilon \to 0 \Rightarrow \left(\frac{d\sigma}{d\Omega}\right)_C \to \left(\frac{d\sigma}{d\Omega}\right)_{Th} = \frac{r_e^2}{2}(1+\cos^2\theta_v)$

And for $\varepsilon >> 0 \Rightarrow \left(\frac{d\sigma}{d\Omega}\right)_C \to 1/\varepsilon$

Compton int prob decreases the higher the photon energy

The higher the photon energy
The more anysotropic the
Scattering (forward)





Inverse Compton Scattering

Most important process in high energy astrophysics, eg when accelerated electrons collide with MWB radiation or other ambient fields

ultrarelativistic electrons scatter low energy photons to high energy. In the reference frame