Grand Unification Theories

After the success of the unified theroy of electroweak interactions it was natural to ask if strong interactions are united with the weak and em ones. The greater strength of strong interactions apparently makes it hopeless but the strength of an interaction depends on the distance over which it acts (the strong coupling constant ‘runs’ much faster with energy than the ew one). In GUT all 3 interactions are united into a single one at the unification mass. The simpliest way to build a GUT (Georgi and Glashow, 1974) incorporates quarks and leptons into common families eg (d_r,d_g,d_b, e^+, $\bar{\nu}_e$) and quarks and leptons can convert into each other. These processes involve the exchange of boson vectors called X, Y with electric charges -4/3 and -1/3 and masses of the order of $M_X \approx 10^{15}$ GeV. GUT predict proton decay (eg $p \to \pi^0 + e^+$ and $p \to \pi^+ + \bar{\nu}_e$) with life time $$\tau \approx \frac{M_X^4}{g_U^4 M_p^5} \approx 10^{30}\, yrs$$ This value can be easily extended since it is very sensitive to M_X .

The hunt for the Higgs

In 1970 a quantum theory that relates week and em interactions (QED) was worked out. In 1982 $W^{\pm}$, Z^0 bosons were discovered at CERN. The masses of these particles where found to be very high (80-90 GeV) in contrast to the massless photon. [The fact that this masses are very large explain the weakness of the interactions since the propagator contains a term $1/M^2$ hence a dependence of the cross section of $1/M^4$].This asymmetry necessitates a symmetry breaking process (Higgs mechanism) not yet proven since a new Higgs field doublet, corresponding to a single neutral Higgs boson, should exist giving mass to W and Z. The present experimental limit on the Higgs mass is $m>115$ GeV (LEP2 reached 209 GeV in 2000).

Higgs found that parameters in the equation describing the field associated to the Higgs particle can be chosen so that the lowest energy state of the field (empty space) is not given by a vanishing field.

The Higgs field has a vacuum expectation value of 250 GeV. The existence of this non null value gives mass to every elementary particle and breaks the electroweak symmetry. The problem is that its mass is not predicted by the theory!

Beyond the Standard Model

SM is extremely successful in describing the 3 fundamental forces: em, weak, strong. Open questions though hint at a more complete theory yet to be found:

- What determines the masses and couplings of SM particles?
- Is there a GUT theory the unifies the 3 coupling constants at ~$10^{15\text{-}16}$ GeV?
- Why the GUT scale and the Planck mass (the scale of gravity unification) >> all SM masses? The contribution to the Higgs boson mass that would come from quantum corrections at the GUT scale, would make Higgs, W, Z masses huge. What prevents them not to be huge?
- Is there a quantum gravity theory?
- How can we unify gravity to the other 3 forces?
- Why matter>antimatter?
- What is dark matter?
- Are quark and leptons elementary and why are there 3 families?

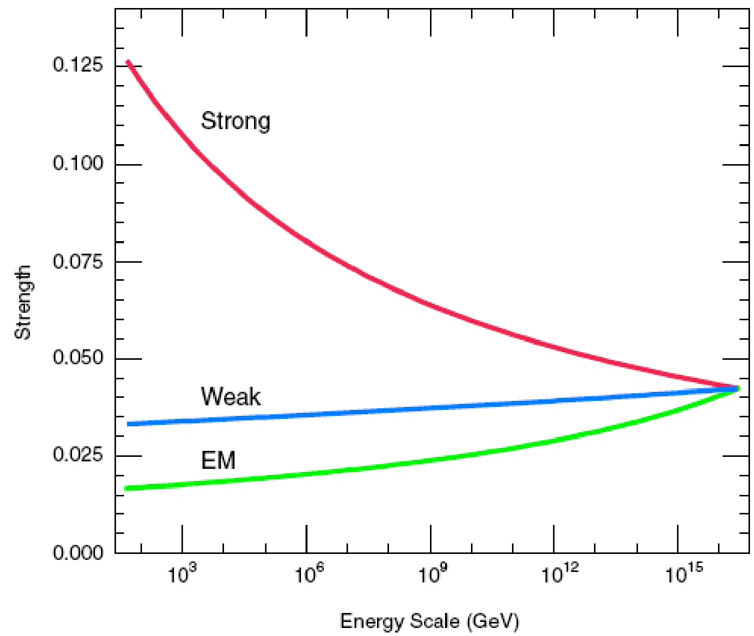


FIGURE 2.4 Variation and convergence of the effective coupling strengths for the three forces—strong, weak, and electromagnetic (EM)—as a function of the energy scale of the interaction. The figure is drawn for a minimal supersymmetric extension of the Standard Model. Without supersymmetry, the three couplings do not precisely meet. Image courtesy of J. Bagger, K. Matchev, and D. Pierce, Johns Hopkins University.

Supersymmetry

It is a symmetry between fermions and bosons.

Photon (spin 1) →photino (spin 1/2)

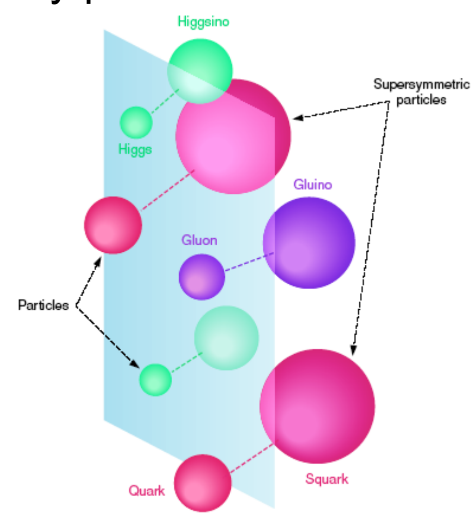
Fermions (spin 1/2) → Sfermions - squarks and sleptons (spin 0)

Z^0, $W^{\pm}$, gluons, Higgs boson (spin 0) → zino, wino, gluino and higgsino(spin1/2)

With supersymmetry the 3 constants unify at 10^{16} GeV

Supersymmetry should be broken since particles and their superpartners must have different masses. In many susy models a conserved quantum number emerges: R-parity which is +1 for ordinary particles and -1 for susy particles

That means that susy particles can be created/annihilated in pairs and that there may be a lightest susy stable particle that can be the dark matter: the neutralino, a linear combination of higgsinos and of the photino and zino



$$\chi = \mathbf{N_1}\tilde{\gamma} + N_2\tilde{Z}^0 + N_3\tilde{H}_1^0 + N_4\tilde{H}_2^0$$

The lower mass limit from colliders is 20-30 GeV

The Minimal Supersymmetric Standard Model requires the existence of 2 Higgs field doublets that provide mass to all charged fermions. There are 2 charged and 3 neutral Higgs bosons. And the lightest is predicted to have m<135 GeV

Neutrinos

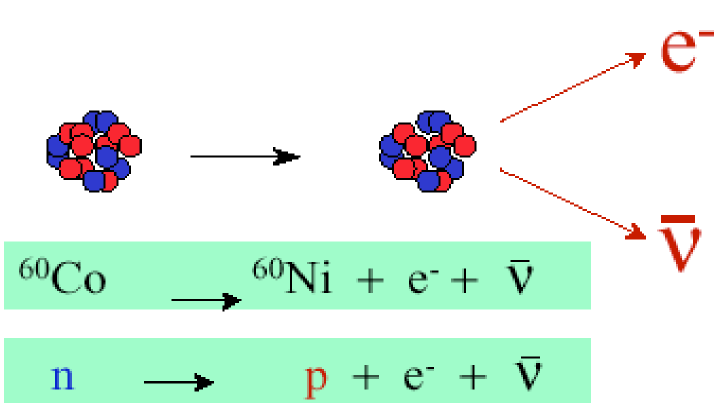
The observation of their oscillations (that implies they have not null mass) have provided first evidence for physics beyond the Standard Model. Similarly the observation of a right handed neutrino or of their decay.

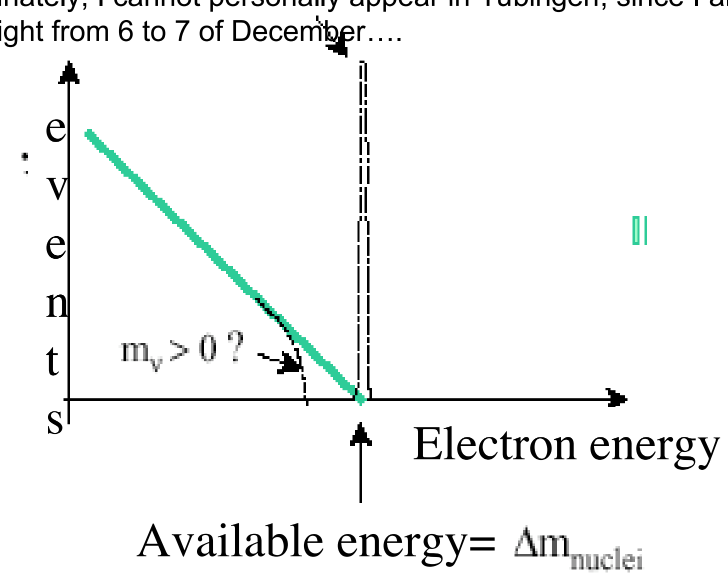
Much of what we know on neutrinos comes from the last 10 years.

In 1930 W. Pauli suggested the existence of an unknown neutral particle of null or very small mass to preserve energy conservation in β-decay since the observed spectrum was continuum

Dear radioactive ladies and gentlemen,

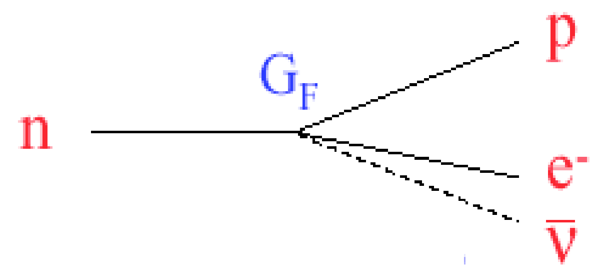
As the bearer of these lines, to whom I ask you to listen graciously, will explain more exactly, considering the 'false' statistics of N-14 and Li-6 nuclei, as well as the the desperate remedy……Unfortunately, I cannot personally appear in Tübingen, since I am indispensable here on account of a ball taking place in Zürich in the night from 6 to 7 of December….



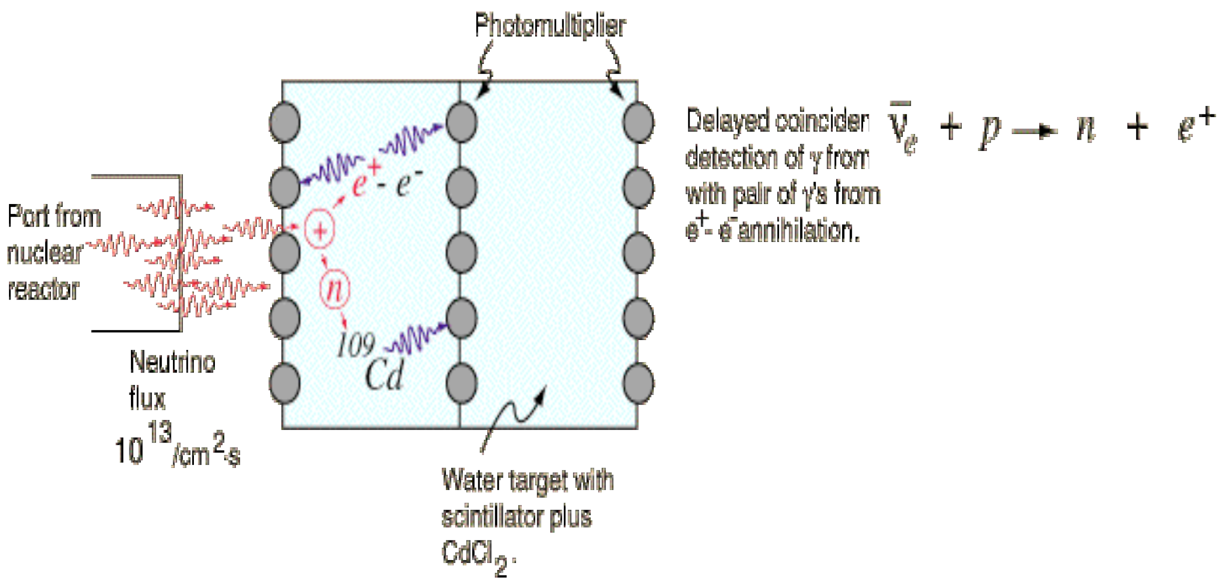


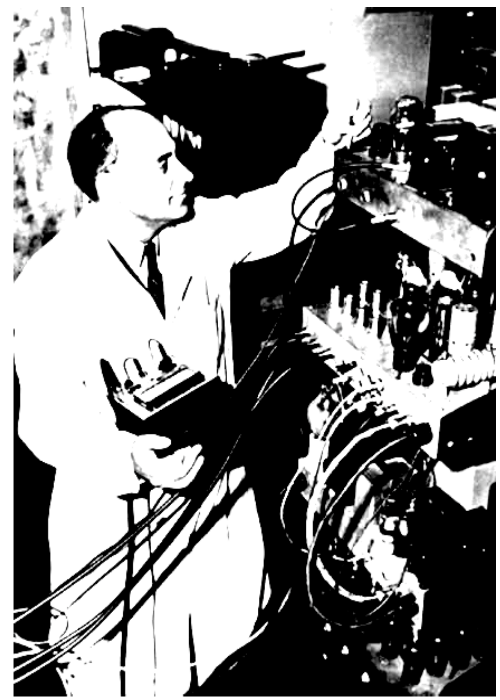
Neutrino History

- 1933 E. Fermi: β-decay teory



- 1956 Cowan and Reines: 1st detection of reactor neutrinos by simultaneous detection of 2γ's from e^+ annihilation and neutron





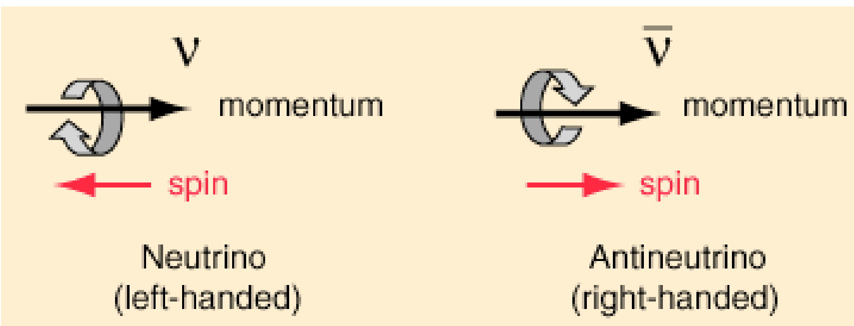
- 1957: Pontecorvo predicts neutrino oscillations

(B. Pontecorvo, Sov. Phys. JETP 6 (1957) 429)

that occur if neutrinos are massive and mixed particles

Is the ν different from the anti-ν?

In SM leptons and quarks are Dirac particles: particle ≠ anti-particle and each has 2 helicity states (left and right-handed) and they obey to Dirac eq. And are described by 4 component spinors. If $m_\nu = 0$, since they are neutral, they are described by Weyl 2 component spinors and travel at c velocity. Neutrinos would be left-handed only and anti-neutrinos right handed.



If neutrinos are massive and Dirac they would behave as electrons: 4 spinors, 2 states for e_L and e_R and 2 for the positron. If an electron is moving along z in a ref frame the spin component along it is -1/2 but one can think to an observer moving in another frame faster than the electron that would look as if the electron moves along -z and so it is right handed. In order to determine who is it (e_R or $\overline{e}_R$) its charge can be measured. But neutrinos are neutral so it would be impossible to distinguish ν_R or $\overline{\nu}_R$. The only observed states are ν_L and $\overline{\nu}_R$, so if it is a Dirac particle also $\overline{\nu}_L$ and ν_R should exist (they can be much more massive).

Neutrino masses

We may avoid introducing these 2 additional states, since the observer cannot distinguish the 2 particles by the charge and hence may be seeing ν_R or $\overline{\nu}_R$.

In the case in which $\nu_R = \overline{\nu}_R$ with violation of total lepton number.

1937 Ettore Majorana theory: the neutrino is the same particle of the anti-ν.

The Dirac-Majorana mass term can be introduced in GUT theories where L is violated.

Through the see-saw mechanism a natural explanation of the smallness of observed ν masses respect to lepton ones arises if the neutrinos we observe are Majorana particles and there exist a much heavier Majorana state

$$m_1 \simeq \frac{(m_D)^2}{m_R} \ll |m_D|, \qquad m_2 \simeq m_R,$$

Since m_D is a Dirac mass term presumably generated with the standard Higgs mechanism it is plausible its value is of the same order f the masses of quarks and leptons of the same generation

ββ-decay

Best way to investigate Majorana/Dirac nature of neutrinos

In SM $\beta\beta 2\nu$ is an allowed process if $M_A(Z,A) > M_A(Z+2,A)$nd the normal b decay is forbidden $M_A(Z,A) < M_A(Z+1,A)$

though very suppressed

$$(Z,A) \rightarrow (Z+2,A) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

Instead $\beta\beta 0\nu$ violates total lepton number conservation

$$(Z,A) \rightarrow (Z+2,A) + e^- + e^-$$

The life-time for this decay is connected to the effective neutrino mass

$$T^{0\nu}_{1/2} \propto \langle m_\nu \rangle^{-2} \qquad |\langle m \rangle| = \left| \sum_k U^2_{ek}\, m_k \right|$$

Present upper limit 0.3-1.3 eV

Neutrino role in the Universe and cosmological bound

The current effort to measure their mass is due to the fact that masses are fundamental constants that need to be measured since they do not come out from the theory.

Neutrinos are extremely abundant in the Universe: the density of neutrinos in the universe is $n_\nu = \frac{3}{11} n_\gamma \approx 110 cm^{-3}$ where n_γ is the current density of photons of the MWB radiation measured by COBE of about 400 γ/cm^3

If neutrinos are Dirac particles (4 states) than it is 220 cm^{-3}

The relic ν contribution to the present density of the universe $\rho_c = \frac{3H^2}{8\pi G_N}$ (H=Hubble constant often expressed as h = 0.71 in units of 100 km s^{-1} Mpc^{-1}, 1 pc = 3.26 ly= 3.1 10^{18} cm, G_N = Newton constant) is given by $\Omega_\nu h^2 = \frac{\sum_k m_k}{94.14 eV}$ (the sum is over the 3 ν flavors and should be multiplied by 2 for Dirac νs). To avoid overclosing the universe the sum of neutrino masses should be < 100 eV and since astronomical data indicate $\Omega_\nu h^2 < 0.1$ and h< 0.8 an upper limit of 6 eV to the sum of masses can be derived. Recent precision data on CMBR (MAP) have strengthened the bound $\Omega_\nu h^2 < 0.0076$ which leads to a limit of 0.71 eV

Another bound from SN1987A

- Exercise: ho can we constrain the neutrino mass from the measured energies and times of events detected in IMB, Kamiokande and other experiments?

Let's pick up 2 of these events

$T_1 = 0$ $E_1 = 20$ MeV

$T_2 = 12.5$ s $E_2 = 10$ MeV

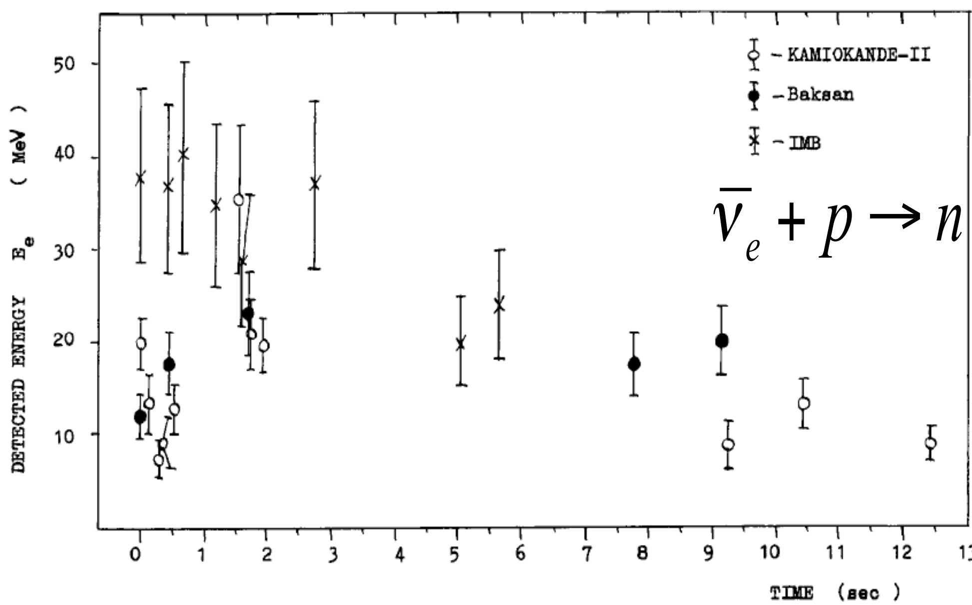
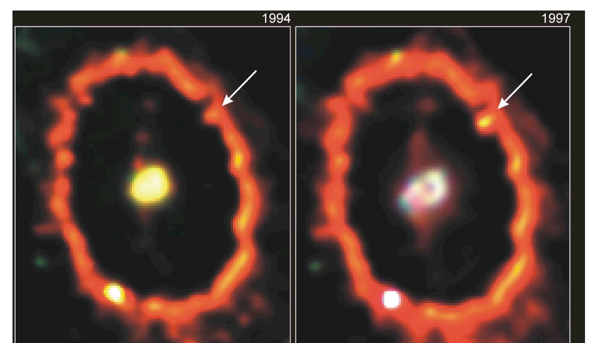


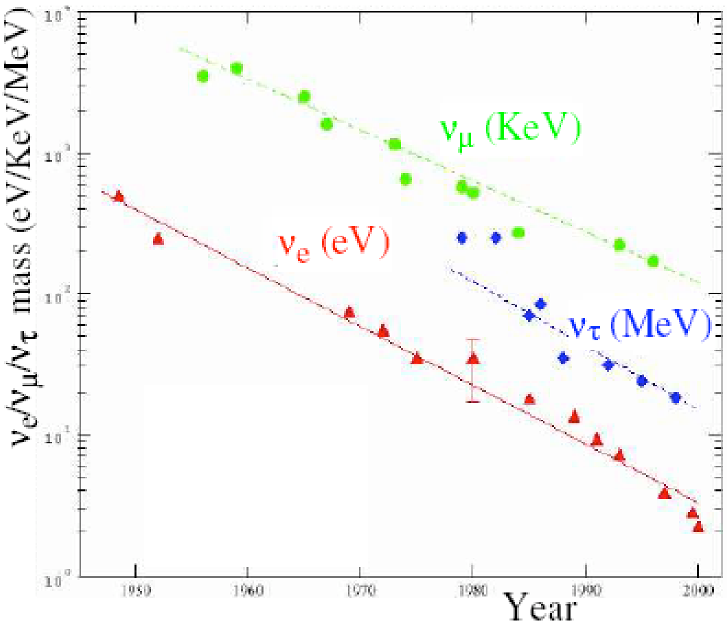
Fig. 3. Energies of all events detected at 7:35 UT on February 23, 1987 versus time. t=0.0 is set as to be the time of the first event of each signal observed.

$$\bar{\nu}_e + p \rightarrow n + e^+$$



Supernova 1987A Ring
Hubble Space Telescope WFPC2

Direct mass limits



- Use $E^2 = p^2c^2 + m^2c^4$ and energy/momentum conservation and no further assumptions
  - m("ν_τ")< 18.2 MeV/c^2 (ALEPH, multi-prong τ decays)
  - m("ν_μ")< 190 keV/c^2 (PSI, muon decay at rest)

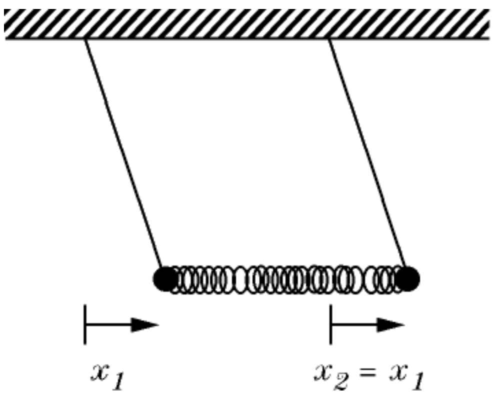
  } not sensitive to the (sub)-eV range

  - m ("ν_e")< 2.2 eV/c^2 (Mainz, tritium beta decay)
- Mass is a property of a stationary state:
  - There is no "ν_e, ν_μ, ν_τ mass"
  - The exact meaning depend on what (and how) is measured

Since masses are so small a convenient way to measure them is ν oscillations though since only squred mass differences are measured and the mixing must be measured too, this requires the combination of many experiments and theory Inputs. This is a world-wide effort!

Neutrinos like coupled oscillators

Two pendula joined by a spring: if one pendulum is started swinging with small amplitude, the other slowly builds up amplitude as the spring feeds energy from the 1st into the 2nd. Then the energy flows back into the 1st and the cycle repeats. A simple situation can be set up for two identical pendula. If you start the two swinging together they will continue to swing in unison at their natural frequency.

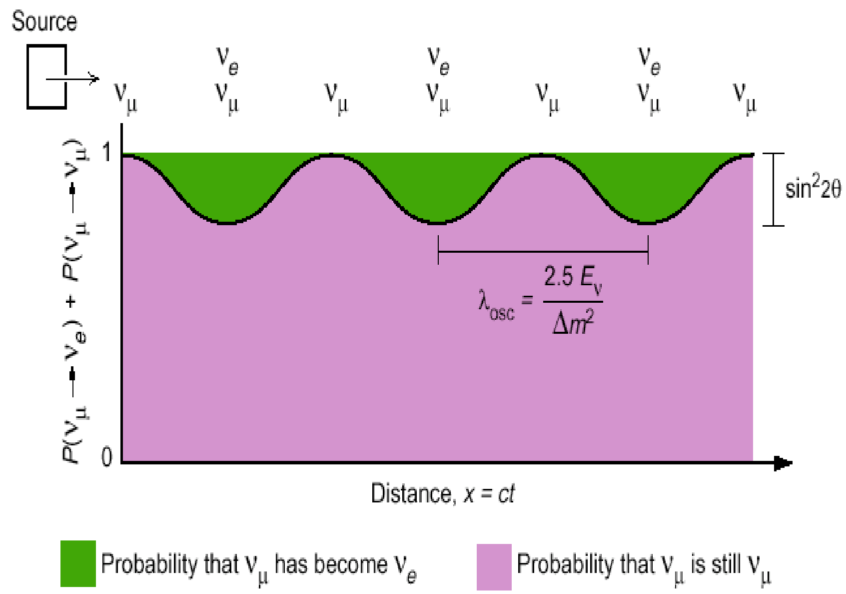


Neutrino oscillations: Pontecorvo's idea

Notice: the neutrino oscillation phenomenon violates the individual flavor lepton numbers not the total one:

$L = L_e + L_\mu + L_\tau$

that is conserved for a Dirac ν and violated in the case of a Majorana one



$$P_{\nu_\mu \to \nu_e} = \sin^2(2\theta)\sin^2\left(1.27\Delta m^2 \frac{L}{E}\right)$$

$$P_{\nu_\mu \to \nu_\mu} = 1 - \sin^2(2\theta)\sin^2\left(1.27\Delta m^2 \frac{L}{E}\right)$$

L in Km, E in GeV, $\Delta m^2 = m_1^2 - m_2^2$ in $(\mathrm{eV}/c^2)^2$

Neutrino oscillations in vacuum

Neutrinos are created and detected in definite flavor states (weak interactions only!) eg in decays with the corresponding lepton $A \rightarrow B + \alpha^+ + \nu_\alpha$

A state of defined flavor is a linear combination of mass eigenstates (states of definite mass)

$$|\nu_\ell\rangle = \sum_\alpha U^*_{\ell\alpha} |\nu_\alpha\rangle$$

A similar process to ν oscillations is found in the system K^0-$\bar{K}^0$ where resulting particles from strong interactions are not physical particles (strangeness eigenstates) but superposition of physical states of different lifetime K_S and K_L. Hence an initial beam of K^0 will evolve as a composition of K^0 and $\bar{K}^0$.

Let's consider a beam of neutrinos of flavor l with momentum p. Since ν_α components have different masses their energies are different:

$$E_\alpha = \sqrt{|\mathbf{p}|^2 + m_\alpha^2}$$

After a time t the evolution of the beam is described by:

$$|\nu_l(t)\rangle = \sum_\beta e^{-iE_\beta t} U^*_{l\beta} |\nu_\beta\rangle$$

And the prob. of finding a ν'_l in a beam originally made of ν_l is:

$$P_{\nu_{\ell'};\nu_\ell}(t) = |\langle \nu_{\ell'} | \nu_\ell(t)\rangle|^2 = \sum_{\alpha,\beta} U_{\ell\alpha} U^*_{\ell'\alpha} U_{\ell\beta} U^*_{\ell'\beta} e^{i(E_\alpha - E_\beta)t}$$

where we used the fact that mass eigenstates are ortonormal $\langle \nu_\alpha | \nu_\beta \rangle = \delta_{\alpha\beta}$

Oscillation probability

If p>>m_α $$E_\alpha = \sqrt{|\mathbf{p}|^2 + m_\alpha^2} \approx p\sqrt{1+\frac{m_\alpha^2}{p^2}} \approx p + \frac{m_\alpha^2}{2p}$$

After a distance x ≈ t for ultrarelativistic νs

$$P_{\nu_\ell \nu_{\ell'}}(x) = \sum_{\alpha\beta} U_{\ell\alpha} U^*_{\ell'\alpha} U_{\ell\beta} U^*_{\ell'\beta} e^{i\left(\frac{2\pi x}{L_{\alpha\beta}}\right)}$$

Where we defined the oscillation length $$L_{\alpha\beta} \equiv \frac{4\pi|\mathbf{p}|}{\Delta m^2_{\alpha,\beta}} = 2.5\frac{p(GeV)}{\Delta m^2_{\alpha,\beta}(eV^2)}(km)$$

If neutrinos were all degenerate $P_{\nu_\ell \nu'_\ell} = \delta_{\ell\ell'}$; and the beam would be always the same.

If neutrinos are Dirac particles the mixing matrix for the 2 family case the mixing matrix is: $$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$ with θ mixing angle

And the flavor eigenstates are

$$\nu_\ell = \nu_1 \cos\theta + \nu_2 \sin\theta$$
$$\nu_{\ell'} = -\nu_1 \sin\theta + \nu_2 \cos\theta$$

$$P_{\nu_\ell;\nu_{\ell'}} = \sin^2 2\theta \sin\left(\frac{\Delta m^2}{4|\vec{p}|}x\right) \quad \ell \neq \ell' \qquad \Delta m^2 = |m_1^2 - m_2^2|$$

$$P_{\nu_\ell;\nu_\ell} = 1 - \sin^2 2\theta \sin\left(\frac{\Delta m^2}{4|\vec{p}|}x\right).$$

if θ =π/4 maximal mixing

The 3 family formalism

The relation between the flavor and mass eigenstates is given by the 3 x 3 matrix: $V = UA$, where A contains the Majorana fase (irrelevant if the ν is Dirac)

$$A = \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & e^{i\beta} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

And U is the MNSP matrix

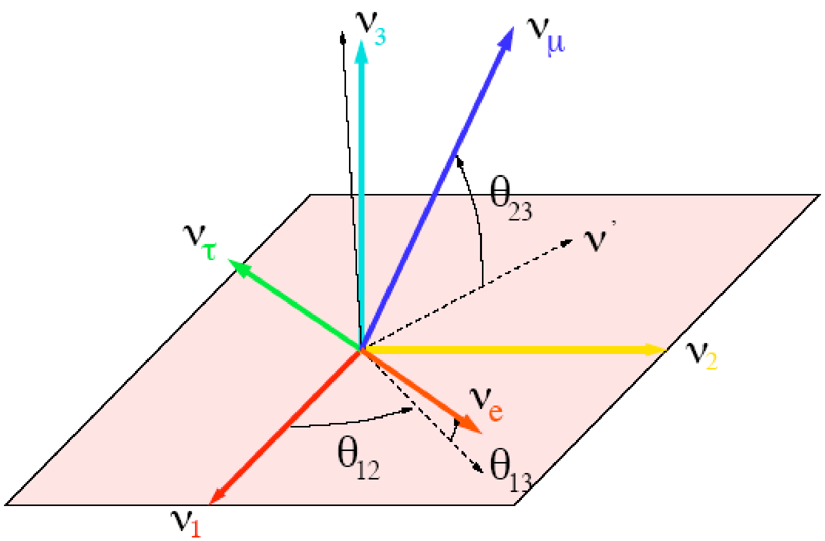
B. Pontecorvo, Sov. Phys. JETP **7**, 172 (1958) [Zh. Eksp. Teor. Fiz. **34**, 247 (1957)].
Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28** 870 (1962)

solar U_{e1}, U_{e2} $\leftrightarrow \theta_{12}$ CHOOZ U_{e3} $\leftrightarrow \theta_{13}$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

atmospheric U_{e3} $\leftrightarrow \theta_{13}$ $U_{\mu 3}$,$U_{\tau 3}$ $\leftrightarrow$ θ_{23}

$$s_{ij} \equiv \sin\theta_{ij},\ c_{ij} \equiv \cos\theta_{ij}.$$



The mixing angles can be derived from matrix elements using:

$$\tan^2\theta_{23} \equiv \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2},$$

$$\tan^2\theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2},$$

$$\sin^2\theta_{13} \equiv |U_{e3}|^2,$$

$$\sin\delta \equiv \frac{8\, Im(U_{e2}^* U_{e3} U_{\mu 2} U_{\mu 3}^*)}{\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}}$$

δ = CP violation phase

The 3 family formalism

Similarly to the 2 family case, the probability of oscillation can be derived for ν:

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sum_{jk} J_{\alpha\beta jk} e^{-i\Delta m^2_{jk} L/2E} \qquad J_{\alpha\beta jk} = U_{\beta j} U^*_{\beta k} U^*_{\alpha j} U_{\alpha k}.$$

For anti-ν: $J_{\alpha\beta jk} \rightarrow J^*_{\alpha\beta jk}$ due to δ. J is not real hence ν and $\overline{\nu}$ probabilities are different ⇒CP is violated in the ν sector.

In a 3 generation scenario full knowledge requires the measurement of:

3 mixing angles $(\theta_{12}, \theta_{23} \text{ and } \theta_{13})$, 2 square mass differences $(\Delta m^2_{12} \text{ and } \Delta m^2_{23})$ and a phase δ. Full knowledge: 9 parameters: 3 masses, 3 mix angles, 3 phases

$$\tilde{J} = c_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} e^{i\delta}.$$

We know: $\Delta m^2_{12} \ll \Delta m^2_{23}$ and θ_{13} small

$$\Delta_{jk} \equiv \Delta m^2_{jk} L/4E$$

$$P_{\nu_e \nu_\mu} = s^2_{23} \sin^2 2\theta_{13} \sin^2 \Delta_{23} + c^2_{23} \sin^2 2\theta_{12} \sin^2 \Delta_{12} + |\tilde{J}| \cos(\delta - \Delta_{23}) \Delta_{12} \sin \Delta_{23}$$

Atmospheric term dominates:
~~θ_{13} large~~ or Δm^2_{12} small

Solar term dominates:
θ_{13} small or ~~Δm^2_{12} large~~

CP violation can be observed if this interference term is separated by the other 2 challenge for future decades

The ν hierarchy

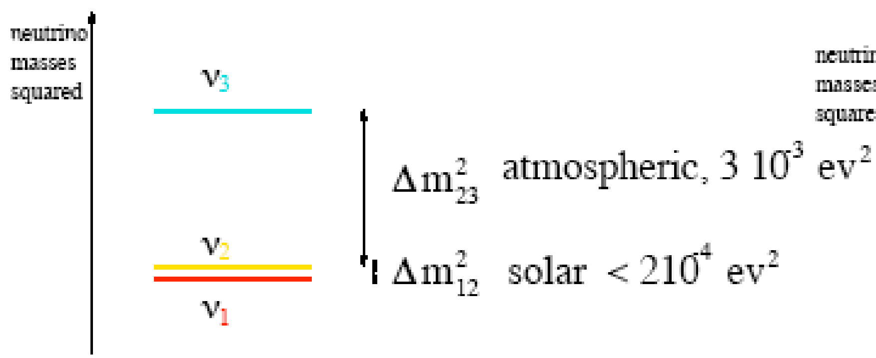
If $\Delta m_{12}^2 = 0$, simplified expressions

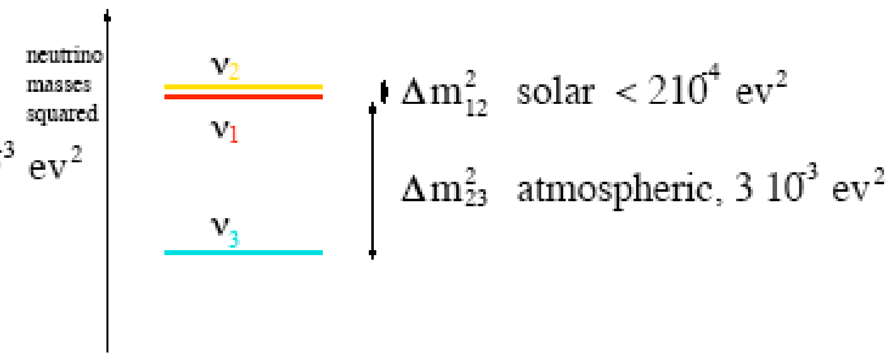
$$
\begin{aligned}
P(\nu_e \to \nu_e) &= 1 - \sin^2 2\theta_{13} \sin^2 \Delta_{23} \\
P(\nu_e \to \nu_\mu) &= \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 \Delta_{23} \\
P(\nu_e \to \nu_\tau) &= \sin^2 2\theta_{13} \cos^2 \theta_{23} \sin^2 \Delta_{23} \\
P(\nu_\mu \to \nu_\mu) &= 1 - 4\cos^2 \theta_{13} \sin^2 \theta_{23} (1 - \cos^2 \theta_{13} \sin^2 \theta_{23}) \sin^2 \Delta_{23} \\
P(\nu_\mu \to \nu_\tau) &= \cos^4 \theta_{13} \sin^2 2\theta_{23} \sin^2 \Delta_{23}.
\end{aligned}
$$

$\Delta m^2_{12} << \Delta m^2_{13} \sim \Delta m^2_{23}$: the 3 family mixing decouples into 2 independent family mixing scenarios

Normal hierarchy: the smallest squared mass difference is generated by the 2 lightest νs

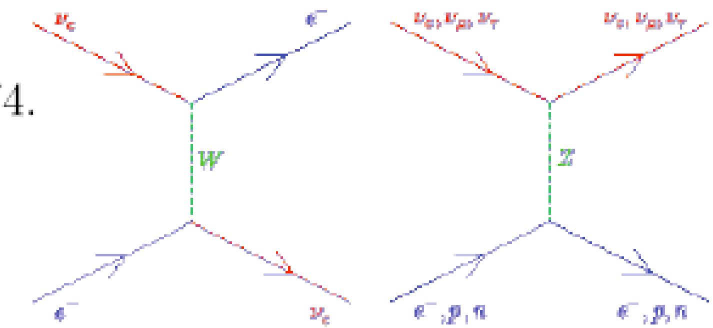
Inverted hierarchy: the smallest Squared mass difference is generated By the 2 heaviest νs





Matter effects

L. Wolfenstein, Phys. Rev. **D17** (1978) 2369-2374.



While ν_e has NC and CC interactions

With the electrons in the media,

ν_μ and ν_τ have only CC int.

The effect can be described by a potential in which ν propagate

$$V_e = \frac{G_F}{\sqrt{2}}[(1+4\sin^2\theta_W)N_e + (1-4\sin^2\theta_W)N_p - N_n] = \frac{G_F}{\sqrt{2}}[2N_e - N_n]$$

$$V_\mu = V_\tau = \frac{G_F}{\sqrt{2}}[(-1+4\sin^2\theta_W)N_e + (1-4\sin^2\theta_W)N_p - N_n] = \frac{G_F}{\sqrt{2}}[-N_n]\,,$$

$N_e = N_p$ matter is neutral

For anti-neutrinos

$$\bar{V}_\alpha = -V_\alpha,\ \alpha = e,\ \mu,\ \tau$$

The same formalism than in vacuum is obtained using $v_{\alpha,\beta} = V_{\alpha,\beta}(2E/\Delta m^2)$

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{[(v_\alpha - v_\beta) - \cos 2\theta]^2 + \sin^2 2\theta} \qquad \ell_m = \frac{\ell}{\{[(v_\alpha - v_\beta) - \cos 2\theta]^2 + \sin^2 2\theta\}^{\frac{1}{2}}}$$

If $v_\alpha - v_\beta = \cos 2\theta$ or $\Delta m^2 \cos 2\theta = 2\sqrt{2}G_F E N_e$ $\Rightarrow$ $\sin^2 2\theta_m = 1$

Mikheyev Smirnov Wolfenstein resonance condition

Experimental Sensitivities

$$P_{\nu_\alpha \to \nu_\beta}(L) = \sin^2 2\vartheta \sin^2 \left(1.27 \frac{(\Delta m^2/\mathrm{eV}^2)\,(L/\mathrm{km})}{(E/\mathrm{GeV})}\right)$$

Appearance or disappearance experiments

Flavor transitions are observables (due to backgrounds and weak interactions that produce low event rates typically if: $\frac{\Delta m^2 L}{4E} \gtrsim 0.1 - 1$.
Hence we can classify based on L/E the range of Δm^2 to which experiments are sensitive:

•Short BaseLine experiments: $L/E \leq 1\ \mathrm{eV}^{-2} \Rightarrow \Delta m^2 \geq 0.1\mathrm{eV}^2$

2 kind of experiments: reactor $\overline{\nu}_e$ disappearance experiments with L≈10m E≈1m (Bugey) and ν_μ accelerators L <~1km and E >~1 GeV (CHORUS $\nu_\mu \to \nu_\tau$ and $\nu_e \to \nu_\tau$, NOMAD $\nu_\mu \to \nu_\tau$ and $\nu_\mu \to \nu_e$, LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ and $\nu_\mu \to \nu_e$ and KARMEN $\bar{\nu}_\mu \to \bar{\nu}_e$ - evidence for sterile neutrino?)

•Long BaseLine and atmospheric neutrino experiments:
low statistics experiments $L/E \leq 10^4\ \mathrm{eV}^{-2} \Rightarrow \Delta m^2 \geq 10^{-4}\mathrm{eV}^2$

Experimental Sensitivities

•Long BaseLine and atmospheric neutrino experiments:

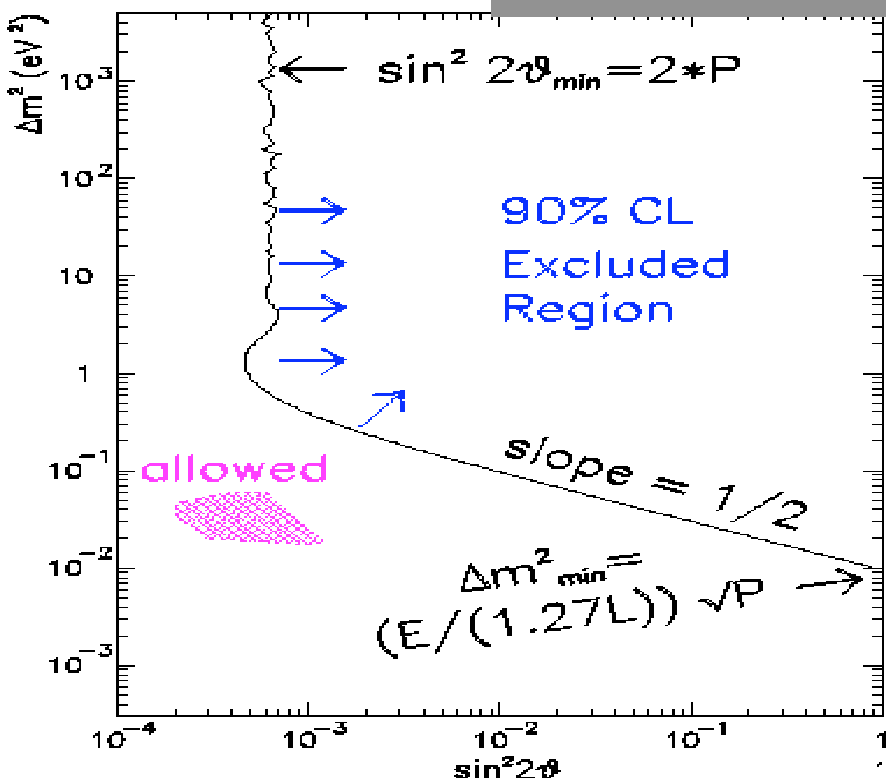
$$L / E \leq 10^4 \ \text{eV}^{-2} \Rightarrow \Delta m^2 \geq 10^{-4} \text{eV}^2$$

2 kind of experiments: reacton $\overline{\nu}_e$ disappearance L ~1km and E >~1 MeV (CHOOZ and Palo verde) and ν_μ accelerators L <~10^3 km and E >~1 GeV (K2K $\nu_\mu \to \nu_\mu$ and $\nu_\mu \to \nu_e$, MINOS $\nu_\mu \to \nu_\mu$ and $\nu_\mu \to \nu_e$ CNGS (OPERA) $\nu_\mu \to \nu_\tau$

Atmospheric neutrino experiments: SK, MACRO, Soudan 2 (L ~20-13000 km E~300 MeV - 100 GeV)

•VLBL reactor experiments (KamLAND L ~180 km) and solar ν experiments (Homestake, Kamiokande, GALLEX and GNO, SAGE, SuperKamiokande)

Oscillation plot



The contour is often determined using the global scan method making a best fit and a χ^2 as a function of $\sin^2 2\theta$ and Δm^2 and the confidence region is given by all points that have $\chi^2 < 4.61\ \chi^2_{min}$ (4.61= 2 sided 90% cl for a χ^2 function with 2 dofs)

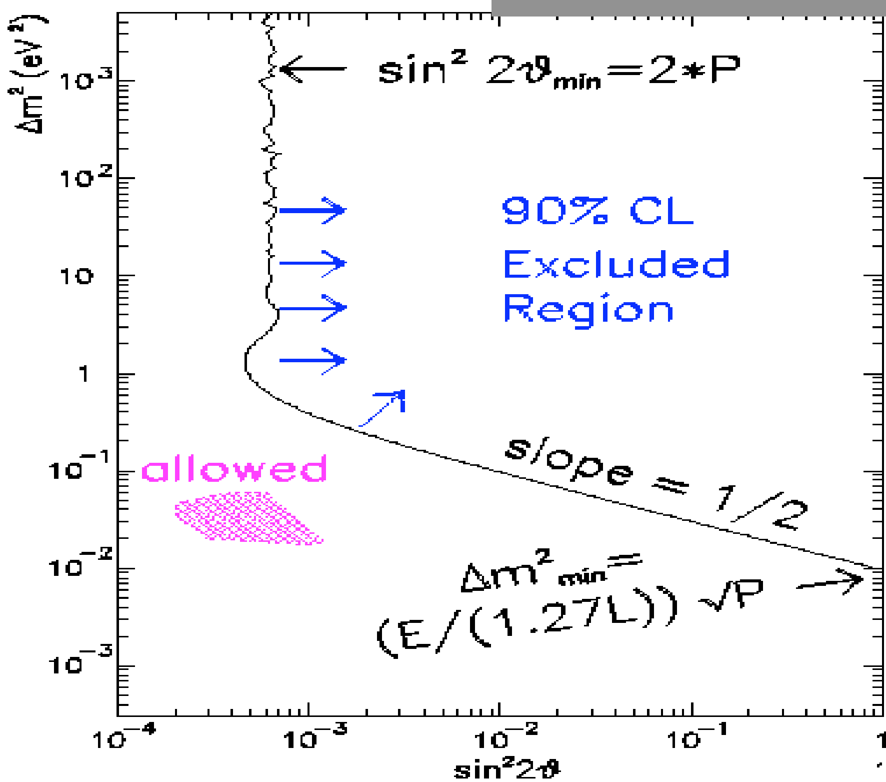
Best method: Feldman Cousins unified approach to calculate two-sided confidence intervals and limits, monte-carlo prescription

Accounts for the fact that $\sin^2 2\theta$ is limited between 0,1

- *large* Δm^2 $Posc = \frac{\sin^2(2\theta)}{2}$
- *small* Δm^2 $Posc = \sin^2(2\theta)\left(1.27\Delta m^2 \frac{L}{E}\right)^2$

The minimum observable value is set by E/L (characteristic of the Experiment by construction)

Oscillation plot



The minimum value of $\sin^2 2\theta$ observable depends on the observable statistics of events:

$$N_{\nu_\ell} \propto (1 - P_{\nu_\ell \nu'_\ell}) \propto (1 - \frac{1}{2}\sin^2 2\theta)$$

$$N_{\nu'_\ell} \propto P_{\nu_\ell \nu'_\ell} \propto \frac{1}{2}\sin^2 2\theta$$

If $\sin^2\theta \ll 1$ K=const

$$\frac{1}{2}\sin^2 2\theta = K\frac{N_{\nu'_\ell}}{N_{\nu_\ell}}$$

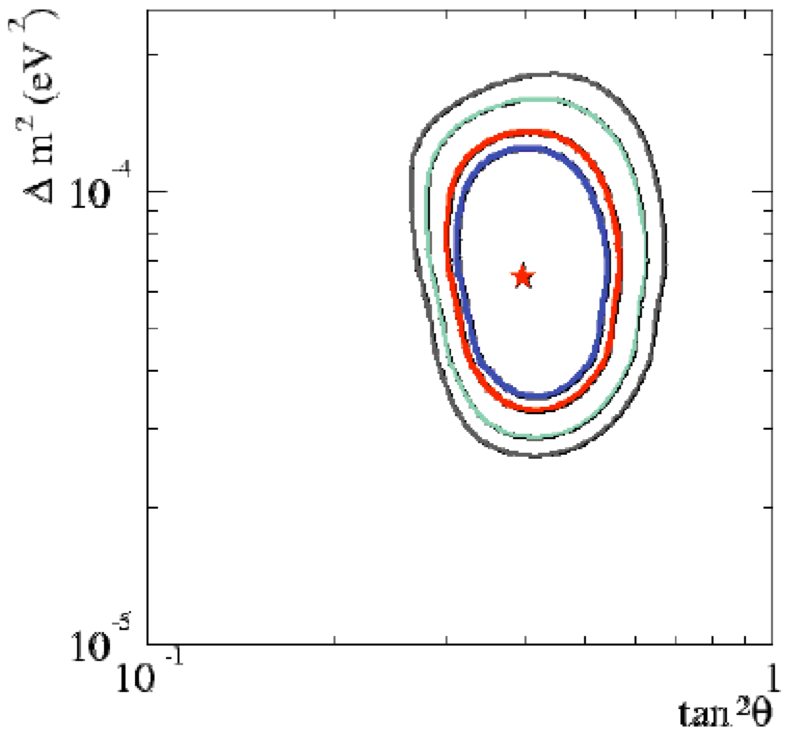
If no event of the other flavor is observed an upper limit can be set

$$\frac{1}{2}\sin^2 2\theta \geq \frac{N_{lim}}{N_{\nu_\ell}}$$

- *large* Δm^2 $Posc = \frac{\sin^2(2\theta)}{2}$ In this case the phenomenon can be observed if the mixing is large enough.
- *small* Δm^2 $Posc = \sin^2(2\theta)\left(1.27\Delta m^2 \frac{L}{E}\right)^2$

Results from oscillation experiments (solar)

$$\Delta m^2_{SUN} = \Delta m^2_{21} << \Delta m^2_{ATM} \approx |\Delta m^2_{31}| \approx |\Delta m^2_{32}|$$



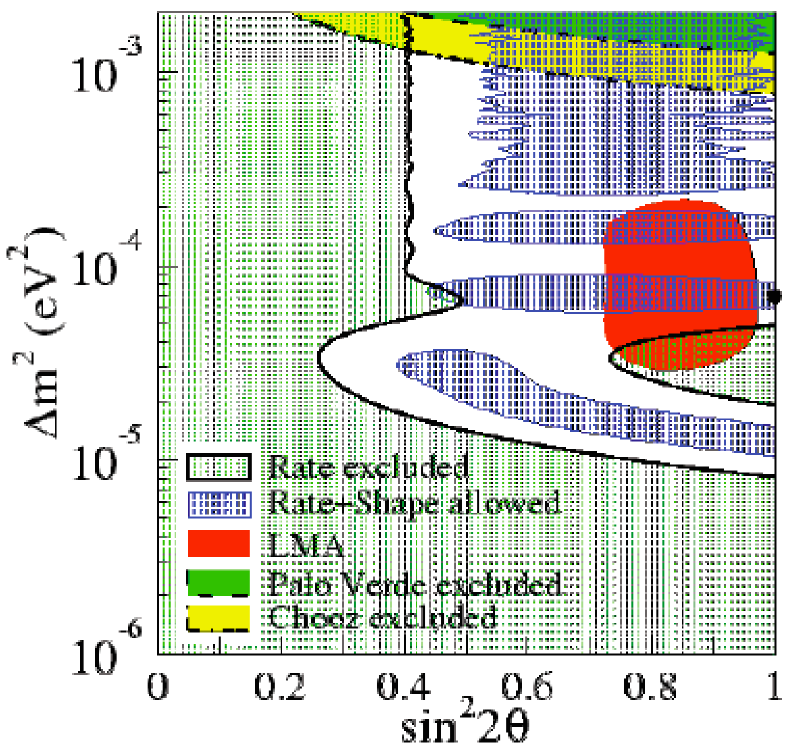


Figure 4: Left: Allowed regions of neutrino oscillation parameters obtained from the global analysis of solar neutrino data [11]. The best-fit point is marked by a star. Right: KamLAND excluded regions of neutrino oscillation parameters for the rate analysis and allowed regions for the combined rate and energy spectrum analysis at 95% C.L [12]. At the top are the 95% C.L. excluded region from CHOOZ [139] and Palo Verde [78] experiments, respectively. The dark area is the 95% C.L. LMA allowed region obtained in Ref. [140]. The thick dot indicates the best fit of KamLAND data.

$5.4 \times 10^{-5}\ \mathrm{eV}^2 < \Delta m^2_{SUN} < 9.4 \times 10^{-5}\ \mathrm{eV}^2$, Best Fit $\Delta m^{2\,bf}_{SUN} = 6.9 \times 10^{-5}\ \mathrm{eV}^2$, $\tan^2 \vartheta^{bf}_{SUN} = 0.43$.

$0.30 < \tan^2 \vartheta_{SUN} < 0.64$,

Results for atmospheric neutrinos

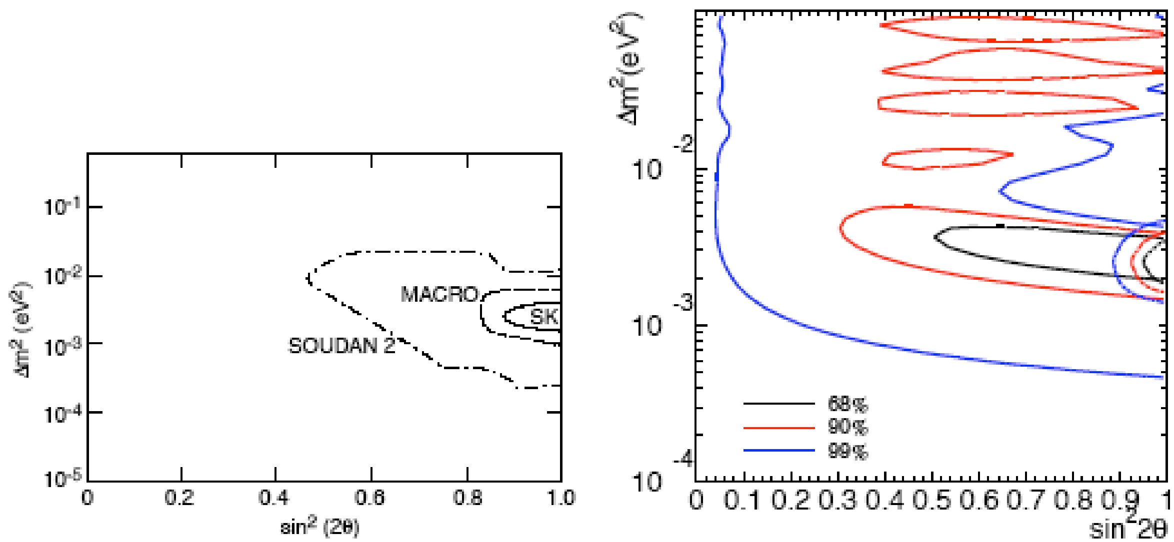


Figure 7: Left: 90% C.L. allowed region contours for $\nu_\mu \to \nu_\tau$ oscillations obtained by the Super-Kamiokande, MACRO and Soudan-2 experiments [29]. Right: Allowed region contours for ν_μ disappearance obtained in the K2K experiment confronted with the allowed regions for $\nu_\mu \to \nu_\tau$ oscillations obtained in the Super-Kamiokande experiment [151].