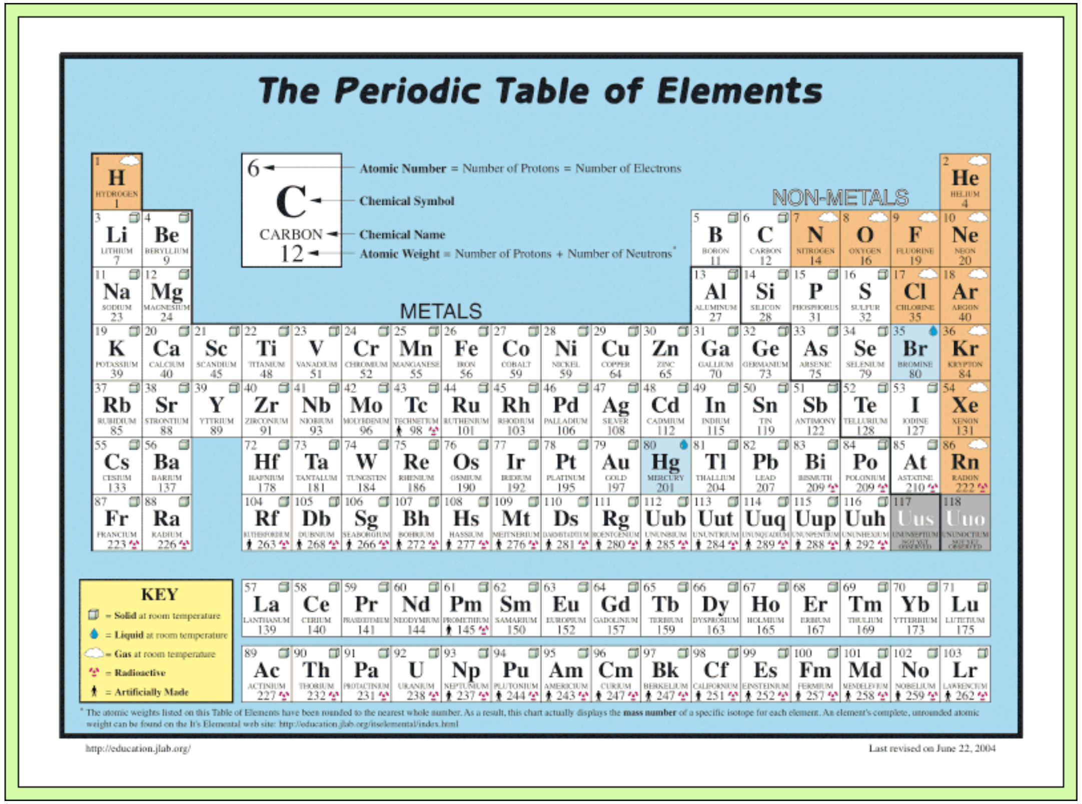
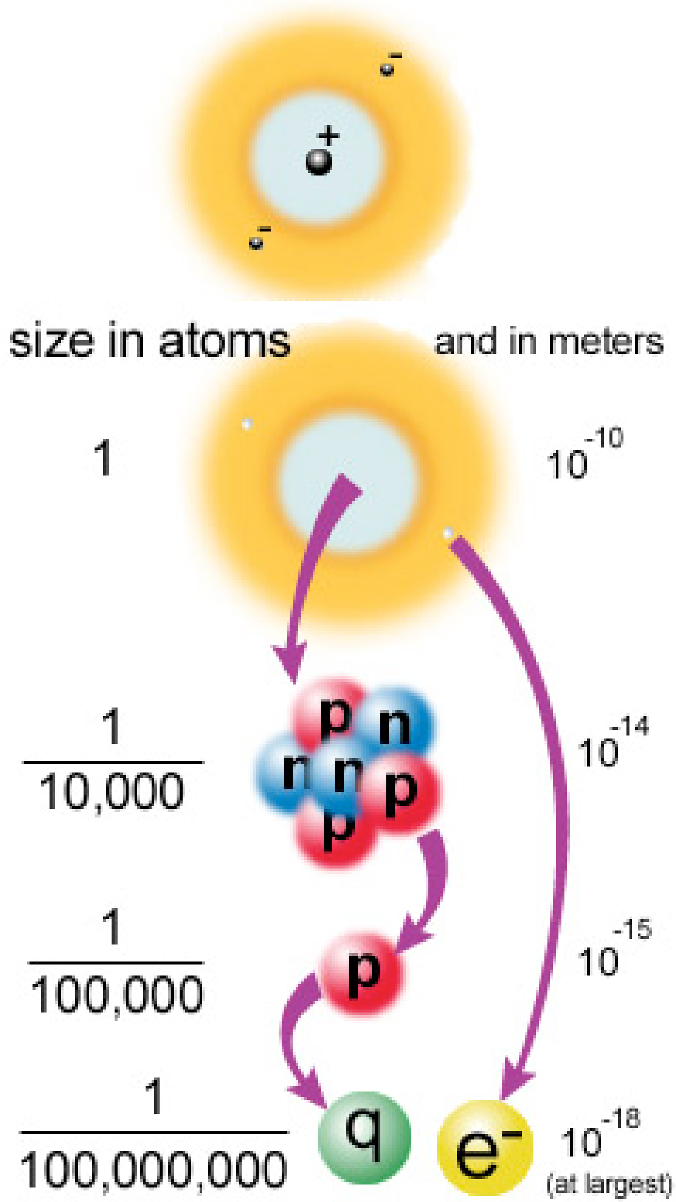
Particles and Fields

- Particles and Antiparticles
- Bosons and Fermions
- Interactions and cross sections
- The Standard Model
- Beyond the Standard Model
- Neutrinos and their oscillations

Particle Hierarchy

Everyday objects are made of molecules
Molecules are made of atoms
Atoms are made of nuclei and electrons
Nuclei are made of protons and neutrons
Protons and neutrons are made of quarks
Quarks and electrons are fundamental?





Macro and Micro worlds

Modern cosmology and particle physics are deeply connected: astroparticle physics is the scientific area that deals with the interplay between the microscopic and macroscopic worlds

Particle physics is the study of the fundamental constituents of matter and their interactions. The modern theory - Standard Model - explains the properties and interactions of the building blocks of matter (they are assumed to be elementary, not made by smaller constituents):

- *quarks and leptons*: fermions of spin angular momentum 1/2 ($\hbar$)
- *gauge bosons*: the force carriers with spin 1 or integer

The spin is a quantum number: fermions/bosons obey the Fermi-Dirac/Bose-Einstein statistics (the wavefunction describing a pair of identical fermions is antysimmetric/symmetric under their exchange - Spin-statistics theorem by Pauli 1940)

$$\Psi \xrightarrow{1\leftrightarrow 2} -\Psi \qquad \text{fermions}$$

$$\Psi \xrightarrow{1\leftrightarrow 2} \Psi \qquad \text{bosons}$$

Consequently: Pauli exclusion principle

2 identical fermions cannot exist in the same quantum state (otherwise their wavefunction should be symmetric) hence on an atomic level there can be 2 electrons with opposite spins

Units and dimensions

1 fm = 10^{-15} m

1 barn = 10^{-28} m^2

1 eV = 1.6 10^{-19} J (1 GeV = 10^9 eV, 1 TeV = 10^{12} eV, 1 PeV= 10^{15} eV, 1EeV= 10^{18} eV)

Since E = mc^2 (rest energy) masses are measured often in MeV/c^2

$\hbar = h/(2\pi)$ = reduced Plank constant =6.582 $\cdot 10^{-25}$ GeV s

$p=h/\lambda$ $E = h\nu$ De Broglie Uncertainty principle $\Delta x \Delta p \geq \hbar$ $\Delta t \Delta E \geq \hbar$

Natural units: $\hbar$ = 1 c = 1

All quantities have the dimension of a power of the energy:

$$M = \frac{E}{c^2} \quad L = \frac{\hbar c}{E} \quad T = \frac{\hbar}{E}$$

Hence a quantity than in MKS has dimensions $M^p L^q T^r$ has dimensions E^{p-q-r}

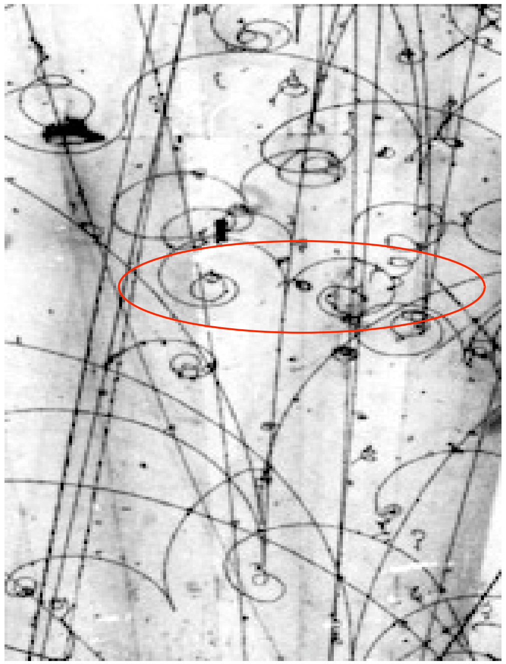
Example: Thompson scattering (photon scattering on free electrons when $E_\gamma \ll m_e c^2$) to have area dimensions

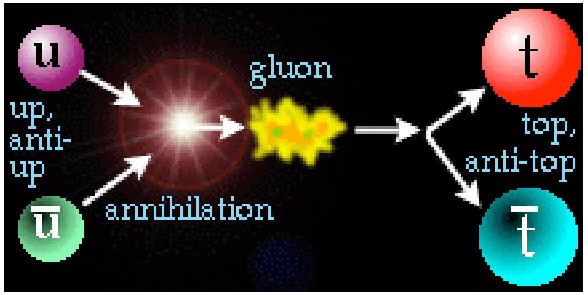
$$\sigma = \frac{8\pi}{3}\frac{\alpha^2}{m_e^2} = \frac{8\pi}{3}\frac{\alpha^2}{m_e^2}\hbar^a c^b = \frac{8\pi}{3}\frac{\alpha^2(\hbar c)^2}{(m_e c^2)^2} = 6.65 \cdot 10^{-29} m^2$$

Matter and Anti-matter

Antiparticles look and behave just like their corresponding matter particles, but have opposite charges. For instance, a proton is electrically positive whereas an antiproton is electrically negative. Gravity affects matter and antimatter the same way because gravity is not a charged property and a matter particle has the same mass as its antiparticle.

When a matter particle and antimatter particle meet, they annihilate into pure energy!





In 1928 Dirac proposed the relativistic equation whose solutions are 4-component wavefunctions corresponding to 2 spin ±1/2 substates of positive and negative energy describing free electrons. Positrons are the states of negative energy. They were discovered in 1933 by Anderson in a cloud chamber experiment with cosmic rays

Matter and Anti-matter asymmetry

Matter is the result of the tiny asymmetry of order of 10^{-9} between matter and antimatter that emerged from the earliest universe. This asymmetry can arise from a baryon non-conserving process.

One of the main mysteries of modern cosmology is what caused this tiny difference: it may be tied to proton decay or to a slight preference for the formation of matter over antimatter such as CP violation C=charge-conjugation operator: transforms a particle in its antiparticle; P = parity = spatial inversion of coordinates

$$P\Psi(\mathbf{r}) \rightarrow \Psi(-\mathbf{r})$$

Theories of Grand Unification foresee p decay with a lifetime of 10^{35} yrs and experimental limits are currently roughly 10^{32-33} yrs.

CP violation, initially observed in the K-meson system and found also in the B-meson system, is allowed in the SM but the magnitude of the violation cannot account for the present asymmetry between matter-antimatter in the universe.

A new source of CP (or T) violation would be needed such as the existence of electric dipole moments of the electron and the neutron (eg a n is uncharged so It could arise from an asymmetry between positive and negative charge clouds relative to the spin direction).

Relativistic wave equations

In quantum theory, a particle with momentum **p** is described by de Broglie wave function

$$\Psi(\mathbf{x},t) = Ne^{i(\mathbf{p}\cdot\mathbf{x}-Et)/\hbar}$$ with N = norm. const. $\nu = E/h$

$\lambda = h/p$

Non relativistically: E = $\mathbf{p}^2$/2m and Ψ obeys the Shröedinger equation obtained substituting the quantum-mechanical operators

$$p \rightarrow -i\hbar\nabla = -i\hbar\frac{\partial}{\partial\mathbf{r}} \qquad i\hbar\frac{\partial\Psi(\mathbf{x},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{x},t)$$

$$E \rightarrow -i\hbar\frac{\partial}{\partial t}$$

Relativistically:

$$E^2 = p^2c^2 + m^2c^4 \Rightarrow -\hbar\frac{\partial^2\Psi(\mathbf{x},t)}{\partial t^2} = -\hbar^2c^2\nabla^2\Psi(\mathbf{x},t) + m^2c^4\Psi(\mathbf{x},t)$$

Klein-Gordon equation for a spin-0 particle

For every solution of momentum **p** and positive energy of the form above there Is a solution of negative energy and momentum -**p**:

$$\tilde{\Psi}(\mathbf{x},t) = \Psi^*(\mathbf{x},t) = Ne^{i(-\mathbf{p}\cdot\mathbf{x}+Et)/\hbar}$$

The Dirac equation

In 1928 Dirac looked for an equation for spin-1/2 particles of the form

$$i\hbar\frac{\partial\Psi(\mathbf{x},t)}{\partial t} = H(\mathbf{x},\hat{\mathbf{p}})\Psi(\mathbf{x},t)$$

$$\hat{\mathbf{p}} = -i\hbar\nabla \qquad \text{hamiltonian}$$

Assuming derivatives in space and time of 1st order (required by Lorentz invariance) the equation becomes:

$$i\hbar\frac{\partial\Psi(\mathbf{x},t)}{\partial t} = H\Psi = -i\hbar c\sum_i \alpha_i\frac{\partial\Psi}{\partial x_i} + \beta mc^2\Psi$$

Where α_i and β are 4 x 4 matrices and Ψ are 4-component wave functions whose values are found imposing solutions of the Dirac are also solution of Klein-Gordon eq.

Plane wave solutions of the Dirac equation are:

$$\Psi(\mathbf{x},t) = u(\mathbf{p})e^{i(\mathbf{p}\cdot\mathbf{x}-Et)/\hbar}$$

where the 4-component spinor satisfies the eigenvalue equation

$$Hu(\mathbf{p}) = \left(c\boldsymbol{\alpha}\cdot\mathbf{p} + \beta mc^2\right)u(\mathbf{p}) = Eu(\mathbf{p})$$

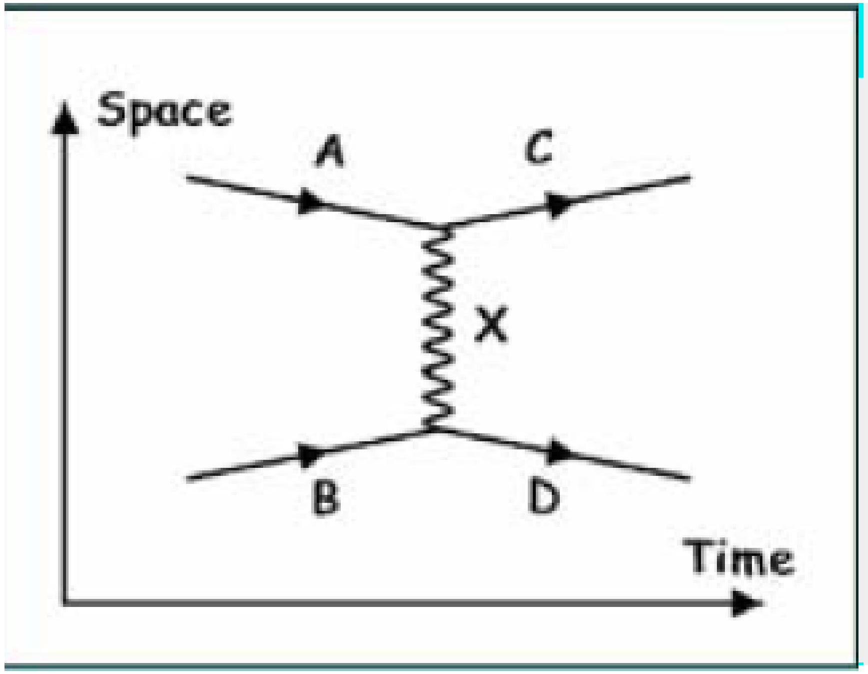
This equation has 4 solutions: 2 of positive energy corresponding to 2 possible states of spin-1/2 particle and 2 negative energy solutions (positron). This results in the prediction that a pointlike spin-1/2 particle of mass m and charge q has a Dirac magnetic moment $\boldsymbol{\mu} = q\mathbf{S}/m$ **S**= spin vector. This provides a test for the point-like nature of any spin-1/2 fermion. For p and n the values indicate that they are not elementary particles

$$\boldsymbol{\mu}_p = 2.79\frac{e}{m_p}\mathbf{S} \quad \boldsymbol{\mu}_n = -1.91\frac{e}{m_n}\mathbf{S}$$

Feynmann diagrams

In 1940 Feynman developed a pictorial technique for describing interactions useful for cross-section calculations. They are representations of the amplitudes of particle reactions (i.e. scatterings or decays, particle exchange). Time flows on the right: an arrow towards the right indicates a particle, towards the left an antiparticle. Each vertex contributes to the amplitude with a constant that represents the strength of the interaction (coupling constant)

X is a virtual particle and the time it exists is governed by the uncertainty principle



Richard Feynman

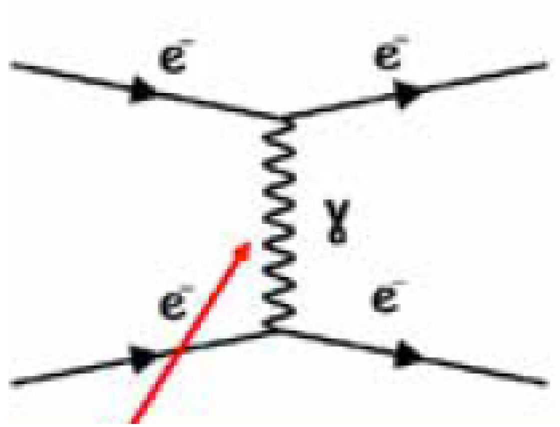
Real processes

Classically: interaction at a distance is described by a field due to a source producing an action in the surrounding region. In quantum theory, quanta (bosons) associated to an interaction are exchanged. Since they carry energy and momentum conservation laws are satisfied only if this exchange occurs in a time limited by the Uncertainty Principle: $\Delta E \cdot \Delta t \leq \hbar$

These transient quanta are then called virtual.

Electromagnetic interactions: the strength of the interaction between charged particles and the photon is given by the fine structure constant

$$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137.0360}$$



In the case of em interactions the exchanged virtual particle is a photon (scattering of 2 electrons)

In the rest frame of the electron

$$\mathbf{e}^-(E_0,\mathbf{0}) \to \mathbf{e}^-(E_k,-\mathbf{k}) + \gamma(ck,\mathbf{k})$$

$$E_0 = mc^2$$ rest energy of e⁻

$$E_{tot} = \sqrt{k^2c^2 + m^2c^4}$$ kinetic E of outcoming e⁻

$$E_{tot} + kc \neq E_0$$

energy cons. violated (but only for a time compatible with uncertainty principle)