Resolution

Another general property of detectors (Hw 7) is the resolution for measuring a quantity Z. If z is the response of the detector to this quantity the resolution is the standard deviation σ_z or the full width at half maximum (FWHM) Δz of the distribution D(z) in the measured quantity z for a monochromatic input distribution δ(Z-<Z>).

The mean value of the measured quantity is $\left\langle \sigma_z^2 \right\rangle = \int \left(z - \langle z \rangle \right)^2 D(z)\, dz$

And the variance is $\sigma_z^2 = \int \left(z - \langle z \rangle \right)^2 dz$

So that the standard deviation is $\sigma_z = \sqrt{\sigma_z^2}$

If D(z) is gaussian the relation between FWHM and standard deviation is

$$\sigma_z = \frac{\Delta z}{2.36}$$

If D(z) is a 'box' distribution (uniform in an interval) the standard deviation is

$\sigma_z = \dfrac{\Delta z}{\sqrt{12}}$ The relative resolution is defined as the dimensionless ratio $\sigma_z / \langle z \rangle$

Resolution

If the only source of fluctuations in the signal is the statistical fluctuation of the number N of primary charge carriers and if their formation follows the Poisson statistics for N>20 the response function is with good approximation a Gaussian distribution and the relative resolution is given by the standard deviation of the Gaussian distribution:

$$\frac{\sigma_z}{\langle z \rangle} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$$

Nonetheless in many types of detectors the resolution can be smaller than this value by a factor up to 4. This indicates that the assumption on the validity of Poisson statistics is false and that there is a correlation between the processes giving rise to the formation of individual charge carriers (Fano effect). So a Fano factor may be introduce to correct for this:

$$F = \left(\frac{\text{observed resolution}}{\text{resolution expected from Poisson statistics}} \right)^2 \approx 0.06(semiconductors) - 1(sc\,int\,illators)$$

TR emission and TR detectors

For a foil of thickness l_1:
Interference produces
a threshold effect in γ

$$\left[\frac{d^2W}{d(h\nu)d\Omega}\right]_{foil} = 4\sin^2\left(\frac{\varphi_1}{2}\right)\frac{d^2W}{d(h\nu)d\Omega}$$

Interference term

$$\varphi_1 \approx \frac{(\gamma^{-2} + Y_1^2)\omega l_1}{2c}$$

Formation zone

For foil thickness << $Z_1(\omega) = \dfrac{2c}{(\gamma^{-2} + Y_1^2)\omega}$ the yield is strongly suppressed

For large ω in order to have radiation emission $l_1 \geq Z_1(\omega) \approx \dfrac{2c\gamma^2}{\omega}$

Radiation can be emitted only for $\gamma > \gamma_{th} = 2.5\omega_p(eV)l_1(\mu m)$
The average TR energy carried by quanta is
For d_1 = 10 μm and ω_1 = 20 eV $\Rightarrow$ $<\omega>$ ~ 3 keV soft X-rays

$$<\omega> \simeq 0.3\,\gamma_{th}\,\omega_1$$

Ability is needed to separate TR from ionization (due to ionization fluctuations and δ rays)

Above threshold the radiation yield increases up to a saturation value
γ_{sat} ~ $\gamma_{th}(l_2/l_1)^{1/2}$ where l_2 is the gap between foils of thickness l_1
When many thin layers of material are joined together the number of photons
can be increased and the limiting factor is reabsorption in the radiator itself.

Typical TR spectrum
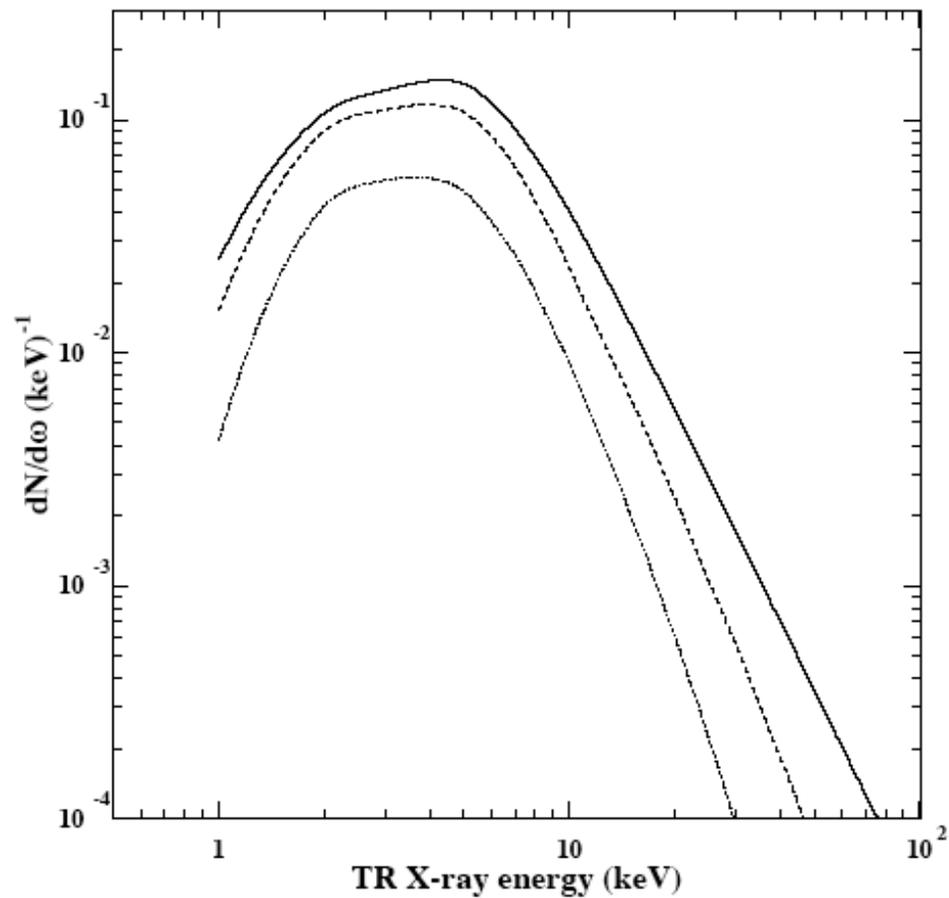
Fig. 1. The TR spectra generated by 250 foils of polyethylene ($d_1 = 5 \ \mu m$ and $\omega_1 = 20 \ eV$) at regular distances $d_2 = 200 \ \mu m$ in air ($\omega_2 = 0.7 \ eV$). Solid line: $\gamma = 5000$; dashed line: $\gamma = 1000$ and dotted line: $\gamma = 500$.

TR detectors

The conventional method for TR detection is the measurement of the sum of the energy released by ionization and photoelectrons produced by TR X-rays. The background of ionization is reduced if gas detectors are used, for example proportional chambers filled with high Z gases, eg Ar or Xe + quenchers to maximize X-ray absorption (photoelectric effect $\propto Z^5$)

The X-ray detector should be thin enough to limit ionization losses of the charged particle. The ionization and excitation is created by a large number of low energy transfers to electrons (δ-rays) producing charge clusters proportional to their energy.

This energy loss is not localized but distributed over the depth of the detector. The absorption of TR produces few local strong energy depositions.

The TR measurement using gas detectors is generally based on 2 methods:

1) Charge measurement: the signal collected from a chamber wire is charge analyzed by ADCs

2) Cluster counting: the wire signal is sharply differentiated to discriminate the X-ray photoelectron clusters producing pulses exceeding a threshold amplitude from the δ-ray ionization background

In both cases a cut is used on the analyzed charge or on the number of clusters to discriminate radiating particles from slower non radiating ones

TR detectors for PID

TR detectors can be used for PID of particles with different masses and same momentum. They can be used for particles with $\gamma>1000$ (below detection of X-rays between 1-5 keV is required)

Eg electrons with p>0.5 GeV/c and pions with p>140 GeV/c

If the pulse height spectrum of a beam containing electrons and pions of 1.4 GeV/c is measured, the tail of the Landau distribution of the pulses generated by ionization of the non-radiating pions can be annoying because it overlaps with the TR radiation region. In this case it is useful to adopt cluster counting methods that reduce the overlap (measuring the total energy in TR and counting ionization clusters along the track that obey a Poissonian distribution with shorter tail than Landau's one)

Electron / pion discrimination

- ● Electrons with radiator
- ○ Electrons without radiator
- ■ Pions with radiator

saturation

ionization

TR
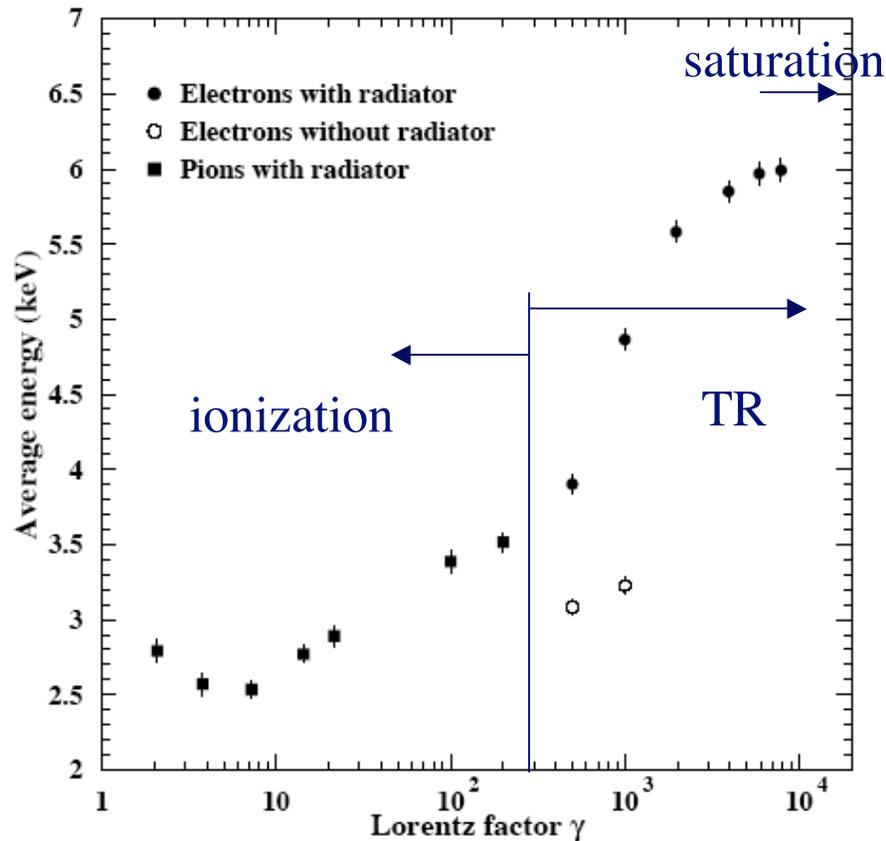
Fig. 8. Average energy loss (summed over 10 planes) as a function of the Lorentz factor. The error bars have been evaluated as ratio of the RMS over the square root of the number of events.

The average energy loss is large for electrons due to the presence of X-ray TR
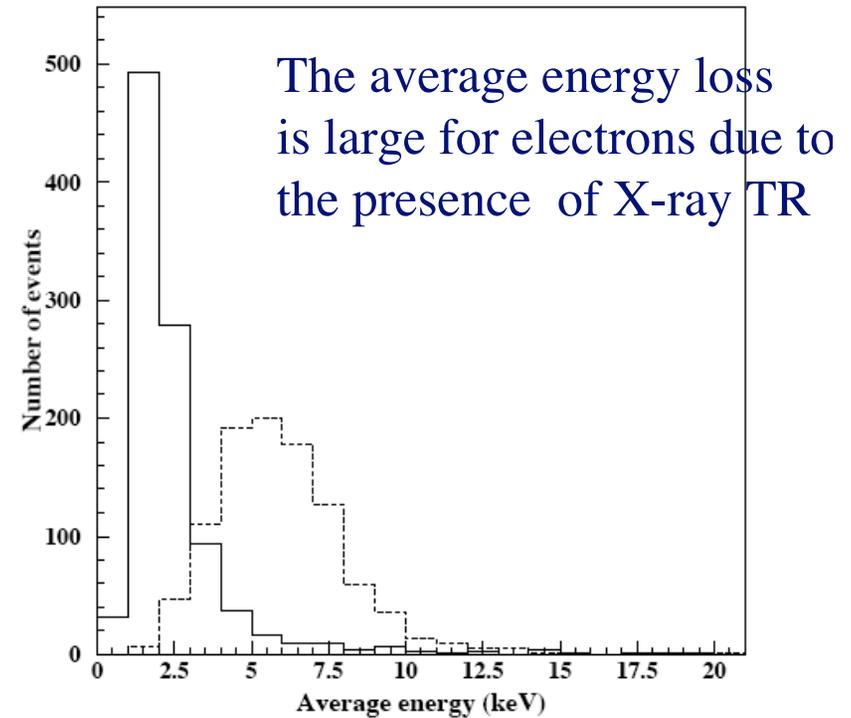
Fig. 9. Average energy loss (summed over 10 planes) distribution for two γ values. Solid line: pions of 255 MeV/c; dashed line: electrons of 4 GeV/c

10 radiators (each with 250 polyethylene foils + chambers of straw tubes (proportional tubes of 2 mm radius)

Cluster size analysis
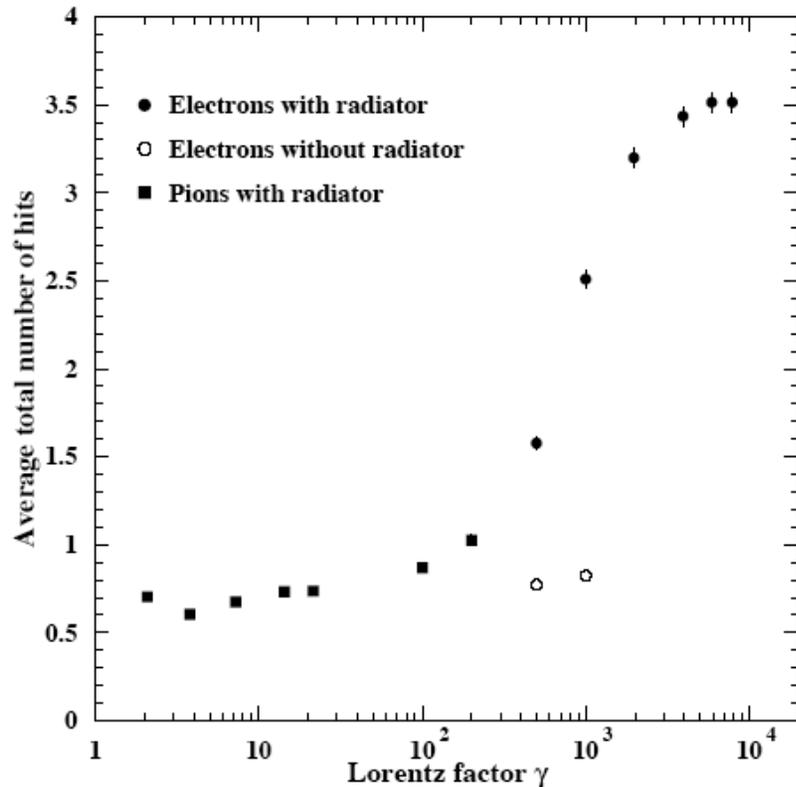
Fig. 11. Total number of hits with a signal greater than 145 mV as a function of the Lorentz factor. The error bars have been evaluated as ratio of the RMS over the square root of the number of events.
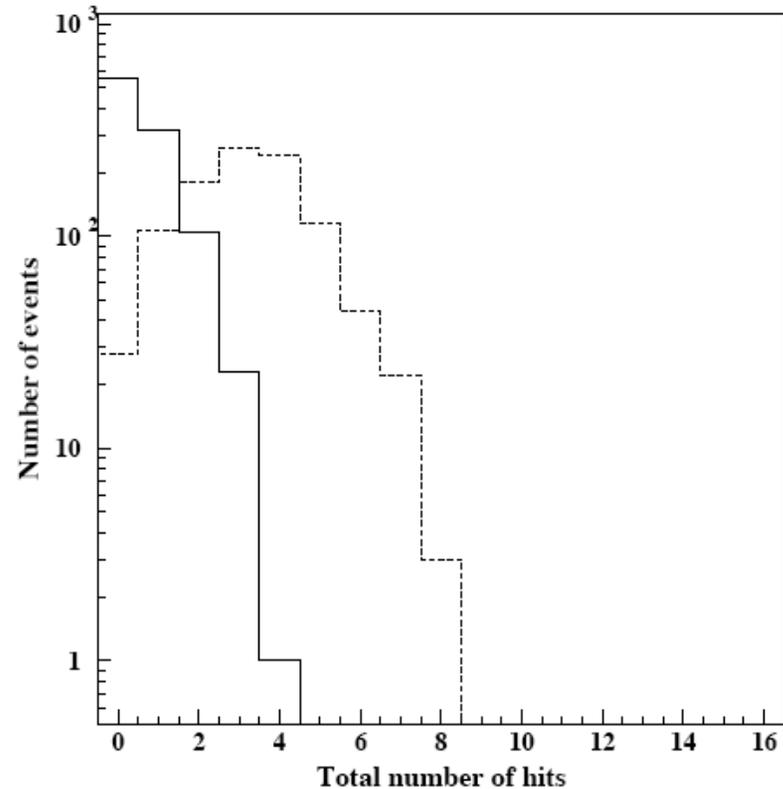
Fig. 12. Hits distribution for two γ values. Solid line: pions of 255 MeV/c; dashed line: electrons of 4 GeV/c

An analysis cut should be decided
Efficiency and contamination

Gas-filled detectors

- A gas-filled detector consists of a volume of gas between two electrodes, with an electrical potential difference (voltage) applied between the electrodes

- Ionizing radiation produces ion-electron pairs in the gas. In gases the ionization energy is typically ~30 eV

- Positive ions (cations) drift towards negative electrode (cathode); electrons (anions) attracted to positive electrode (anode) and induce signal current on the electrodes

- In most detectors, the cathode is the wall of the container that holds the gas and the anode is a wire inside the container
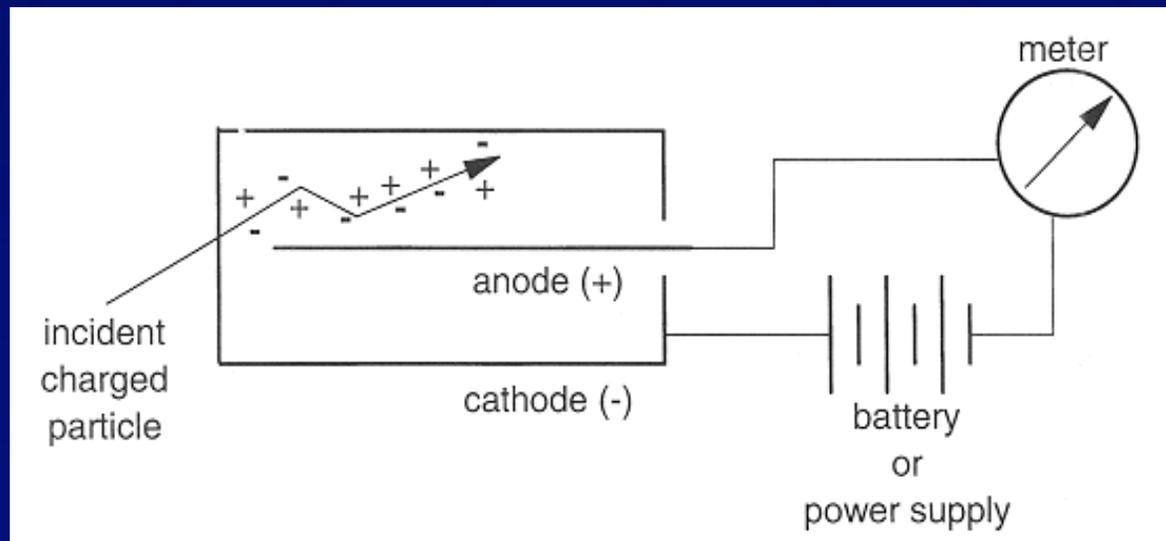
Drift of electrons

Electrons undergo random collisions when drifting in a gas. If v is the velocity and τ the average time between collisions, their number in a distance dx is:

$$dn = \frac{1}{v\tau}dx$$

and the collision rate in a gas of density N is

$$\frac{1}{\tau} = N\sigma v'$$

where v' is the instantaneous velocity. Between 2 collisions the electrons is accelerated by the electric field:

$$m\frac{dv}{dt} = q_e E$$

and the electron displacement vs time is

$$x(t) = \frac{1}{2}\frac{q_e E}{m}t^2$$

Since the differential probability of collisions is

$$dP = \frac{1}{\tau}e^{-t/\tau}dt$$

the average displacement is

$$\langle x \rangle = \int_0^\infty \frac{1}{2}\frac{q_e E}{m}t^2\frac{1}{\tau}e^{-t/\tau}dt = \frac{q_e E}{2m}\left\{\left[-e^{-t/\tau}t^2\right]_0^\infty + \int_0^\infty 2te^{-t/\tau}dt\right\} = \frac{q_e E}{m}\left\{\left[-\tau e^{-t/\tau}t\right]_0^\infty + \int_0^\infty \tau e^{-t/\tau}dt\right\} = \frac{q_e E}{m}\tau^2$$

The average drift velocity is

$$\langle v \rangle = \frac{\langle x \rangle}{\tau} = \frac{q_e E}{m}\tau = \mu E$$

μ = mobility

Since $\tau \propto 1/N \propto 1/p$ the velocity is commonly written

where the mobility is determined at standard pressure

p_0 =760 torr

$$v = \mu E \frac{p_0}{p}$$

Drift of electrons

This simple relation does not hold everywhere since the collision cross-section is not constant with energy. At around 0.1 eV the cross
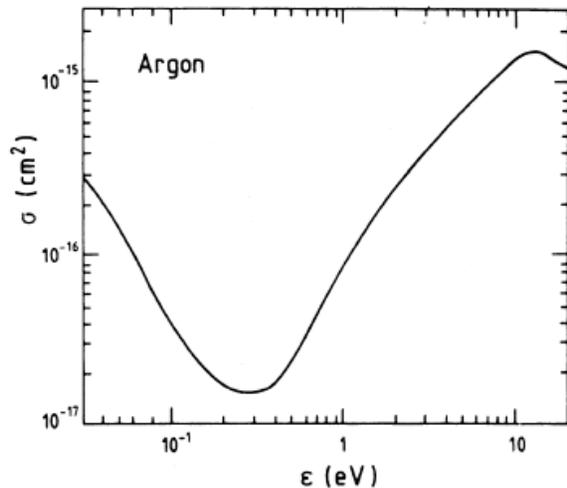
section has a min (Ramsauer effect)

and the gas is almost transparent to electrons, so acceleration is large but then it decreases (the cross section increases)

Electrons can be

captured by

electronegative gases

O_2, Cl_2^-, NH_3 and H_2O

leading to loss of signal

Electron velocity vs. E/p for various mixtures of Ar and CH_4.

The dashed line is a calculation.
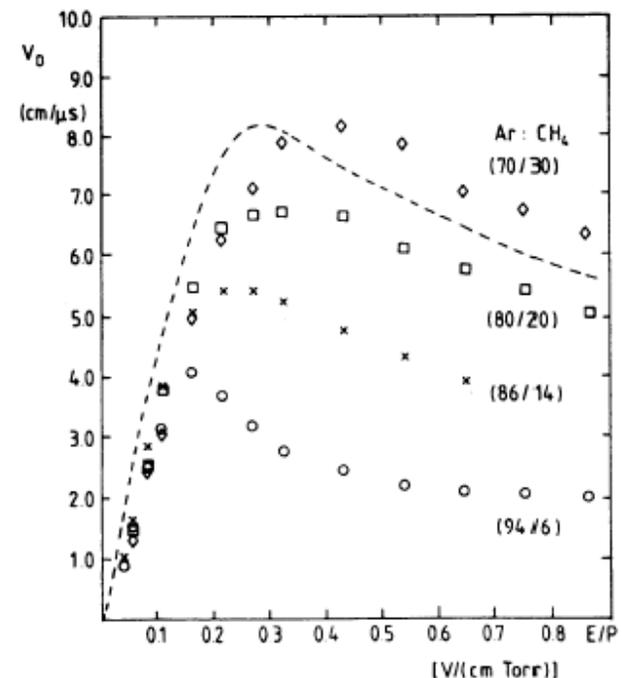
Drift of ions

A similar expression holds for ions
$$v^+ = \mu^+ E \frac{p_0}{p}$$

Measured mobilities of ions in various gases

Gas	Ion	Mobility [cm²/Vs]
He	He$^+$	10.2
Ar	Ar$^+$	1.7
Ar	$(OCH_3)_2CH_2^+$	1.51
Iso-C_4H_{10}	$(OCH_3)_2CH_2^+$	0.55
$(OCH_3)_2CH_2$	$(OCH_3)_2CH_2^+$	0.26
Iso-C_4H_{10}	$IsoC_4H_{10}^+$	0.61
Ar	CH_4^+	1.87
CH_4	CH_4^+	2.26
Ar	CO_2^+	1.72
CO_2	CO_2^+	1.09

At 1 kV/cm ions are of order 1000 times slower than electrons.

Parallel plate ionization chambers

In these chambers the ionization produced by a passing particle is measured
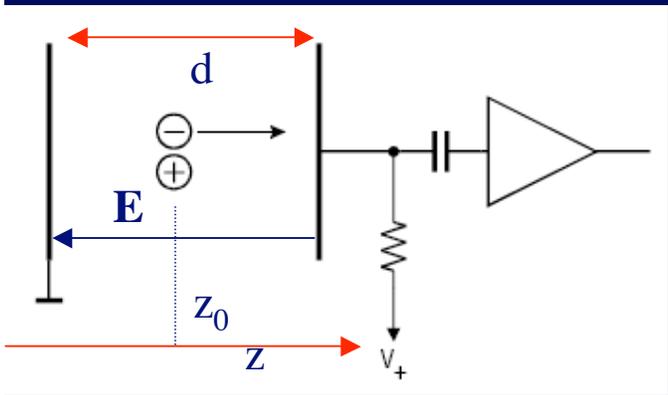
The simplest form is a parallel plate capacitor filled with gas. The value of the electric field $E_z = U_0/d$ must be such that N positive and N negative charges formed along the path of the particle passing at $z=z_0$ are completely collected on the electrodes but no secondary ionization occurs.

The charge produced at z_0 flows towards the electrodes and induces a voltage pulse. For energy conservation:

$$\frac{1}{2}CU^2 = \frac{1}{2}CU_0^2 - N\int_{z_0}^{z} qEdr \Rightarrow \frac{1}{2}C(U+U_0)(U-U_0) \approx \frac{1}{2}C2U_0\Delta U = -Nq\frac{U_0}{d}(z-z_0) \Rightarrow$$

$$\Delta U = -\frac{Nq}{Cd}(z-z_0)$$

$$\Delta U^+ = -\frac{Ne}{Cd}v_D^+\Delta t^+$$

$$\Delta U^- = -\frac{N(-e)}{Cd}(-v_D^-)\Delta t^-$$

For a constant drift velocity of ions/electrons: the polarity of the 2 contributions to the pulse is the same

Pulse shape

Since electrons drift faster than ions, the pulse increases first due to electrons up to a value $\Delta U = -\frac{Ne}{Cd}z_0$ up to the asymptotic value due to ions $\Delta U = -\frac{Ne}{C}$

Given a voltage of 1 kV over an electrode spacing of 1 cm (E = 10^3 V/cm) the electron drift velocity is typically 10^6 cm/s and the ion one is 10^3 cm/s. So electrons/ions require 1 µs/ms to traverse the detector. If the integration time of the signal is 1 µs the induced charge is

$$\Delta Q = Nq_e \frac{\Delta x}{d} = Nq_e \frac{vt_{int}}{d} = Nq_e \frac{10^3 \cdot 10^{-6}}{1} = 10^{-3}Nq_e$$
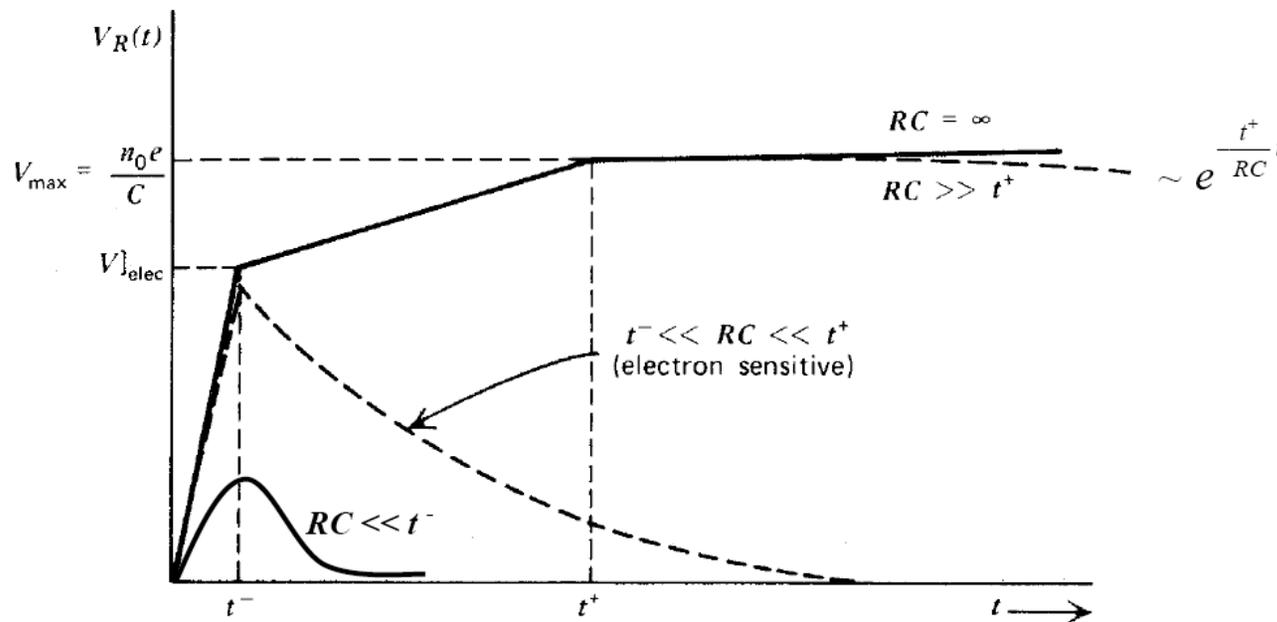
Fig. 3-5. Output pulse shape for the various time constant in the schematic diagram of Fig. 3-4.

Parallel plate ionization chambers

The voltage pulse is independent of z_0 if RC>>Δt^+ of ions (too long for collecting single pulses!)
If only the electron signal is taken RC<<Δt^- the signal is nearly independent on z_0 but the signal is small.
For Δt^- << RC<<Δt^+ the signal depends on the position z_0. The position dependence of the induced charge can be removed using a shielding grid: charges moving in the space d_1 induce current on the left electrode and on the grid but not on the signal electrode. Electrons drift towards the grid, traverse it and move in d_2 towards the grid. If the pulse is collected between the grid and the anode, all electrons have the same drift time between the grid and the anode and so all electrons contribute the same amount to the pulse and the pulse amplitude is independent of position
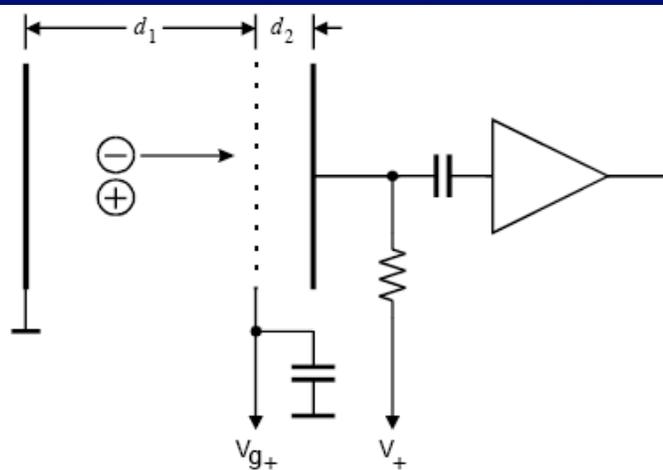
Proportional chambers

If the electric field is high enough, electrons acquire sufficient energies between collisions to ionize gas molecules. This can be exploited to increase the signal. The amount of charge liberated in the process can be amplified by a factor G=gas amplification factor and the voltage pulse is proportional to the primary ionization charge

$$\Delta U = -G \frac{Ne}{C}$$

The probability of ionization is determined by the Townsend coefficient α: the increase in the number of electrons traversing dx is: $dN = N \alpha \, dx$

So the gain in a uniform electric field is over a distance L (if α = const)

$$G = \frac{N}{N_0} = e^{\alpha L}$$

High fields are often obtained using the radial dependence of the electric field near a thin wire (diameters 25-50 μm).

$-U_0$
$2r_1$
R
$2r_2$
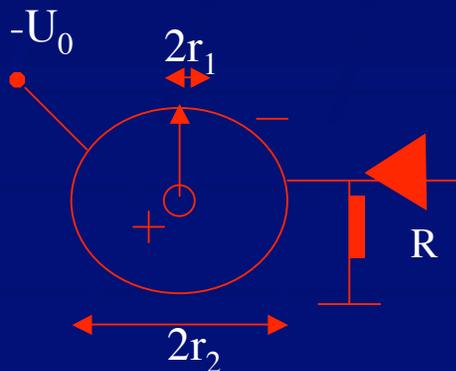
Proportional chambers

In general α depends on the electron energy and therefore on the electric field strength

$$G = \frac{N}{N_0} = \exp\left(\int \alpha(r)dr\right) \Rightarrow \ln G = \int \alpha(r)dr = \int_{E(r_c)}^{E(r_1)} \alpha(E)\frac{dr}{dE}dE$$

In noble gases $\alpha(E) \approx \beta E$ so that G rises exponentially with U_0

The electric field can be determined using the Gauss law:

$$E(r)2\pi rL = Q/\varepsilon_0 = \lambda L/\varepsilon_0$$

$$E = -\frac{dV}{dr} \Rightarrow \int_{r_1}^{r_2} Edr = -\int_{r_1}^{r_2} dV \Rightarrow V(r_1) - V(r_2) = \int_{r_1}^{r_2} \frac{\lambda}{2\pi\varepsilon_0 r}dr \Rightarrow U_0 = V(r_1) - V(r_2) = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_2}{r_1}$$

$$U_0 = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_2}{r_1} \Rightarrow E(r) = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{U_0}{r\ln\frac{r_2}{r_1}}$$

$-U_0$ $2r_1$

R

$2r_2$
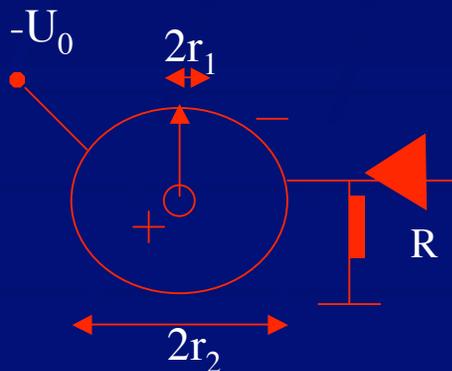
Proportional chambers

The proportional counter is a condenser of capacity C. If r_c is the radius at which the avalanche settles The charge produced at r_0 flows towards the electrodes and induces a voltage pulse. For energy conservation:

$$\frac{1}{2}CU^2 = \frac{1}{2}CU_0^2 - N\int_{r_0}^{r} qEdr \Rightarrow \frac{1}{2}C(U+U_0)(U-U_0) \approx \frac{1}{2}C2U_0\Delta U = -Nq\int_{r_0}^{r_2}\frac{U_0}{r\ln\frac{r_2}{r_1}}dr \Rightarrow$$

$$\Delta U^+ = -\frac{Ne}{C}\frac{\ln\frac{r_2}{r_0}}{\ln\frac{r_2}{r_1}}$$

Where we wrote the electric field as:

$$U_0 = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_2}{r_1} \Rightarrow E(r) = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{U_0}{r\ln\frac{r_2}{r_1}}$$

Similarly for ions:

$$\Delta U^- = -\frac{Ne}{C}\frac{\ln\frac{r_0}{r_1}}{\ln\frac{r_2}{r_1}}$$

For $r_2/r_1 = 10^3$ and $r_0 = r_2/2$
$|\Delta U^+/\Delta U^-|\sim 10$
The total charge is dominated by ions, but the electron current is larger due to the greater mobility of electrons but very fast electronics is needed to use it. The long drift time of ions limits the rate capability of proportional counters. The use of small gas volumes increase rate capability (eg straw tubes allow rates of 10^7 s^{-1})

Types of gas-filled detectors

Three types of gas-filled detectors are commonly used:

- Ionization chambers
- Proportional counters
- Geiger-Mueller (GM) counters

Type determined primarily by the voltage applied between the 2 electrodes

E_c critical field required for the onset of appreciable ionization

ε_i ionization energy

Gas mixture	$E(r_c)$ [kV/cm]	ε_i [Volts]
90% Ar + 10% CH$_4$	48 ± 3	23.6 ± 5.4
95% Ar + 5% CH$_4$	45 ± 4	21.8 ± 4.4
92.1% Ar + 7.9% CH$_4$	47.5	30.2
23.5% Ar + 76.5% CH$_4$	196	36.2
9.7% Ar + 90.3% CH$_4$	21.8	28.3
0.2% Ar + 99.8% CH$_4$	171	38.3
CH$_4$ (Hendricks)	69 ± 5	36.5 ± 5.0
CH$_4$ (Diethorn)	144	40.3
C$_3$H$_8$	100 ± 4	29.5 ± 2.0
75% Ar + 15% Xe + 10% CO$_2$	51 ± 4	20.2 ± 0.3
90% Xe + 10% CH$_4$	36.2	33.9
95% Xe + 5% CO$_2$	36.6	31.4

Gas fillings typically consist of noble gases (eg Ar, Xe) because energy dissipation in these gases mainly goes into ionization, de-excitation photon energy > ionization threshold of metals that are used for electrodes
The average energy for creating an electron -ion pair is 41 eV for He, 22 eV for Xe, 11 eV for Ar. In semiconductors this energy is much less, 3.5 eV in Si and 2.85 eV in Ge hence the number of electron-hole pairs is much larger for the same energy deposition and semiconductor counters can achieve better E resolution

Gas mixtures and quenching

The discharge process is stopped by the cloud of positive ions near the anode wire. These ions reduce the electric field around the wire so that further electrons arriving there cannot form avalanches. When ions drift towards the cathode the process can start again. The discharge can be quenched externally reducing the voltage temporarily or by choosing the external resistor R large so that the voltage drop due to the anode current reduces the instantaneous anode potential or by using a quenching agent (C_2H_5OH, CH_4, isobutane C_4H_{10}, methylal $(OCH_3)_2CH_2$).

These molecules can absorb UV quanta produced in de-exicitations during the avalanche process and they can reduce the range of these photons so that they do not reach the cathode and do not produce photoelectrons. Moreover the creation of secondary electrons by positive ions is reduced because the ions of the counting gas transfer their ionization to the quenching gas, but these do not get enough for sizeable secondary ionization.

Gas amplification vs applied voltage

Ionization mode: full charge collection, no amplification

Proportional mode:

Multiplication

Signal proportional to primary ionization

Quenching

Geiger mode

Full discharge
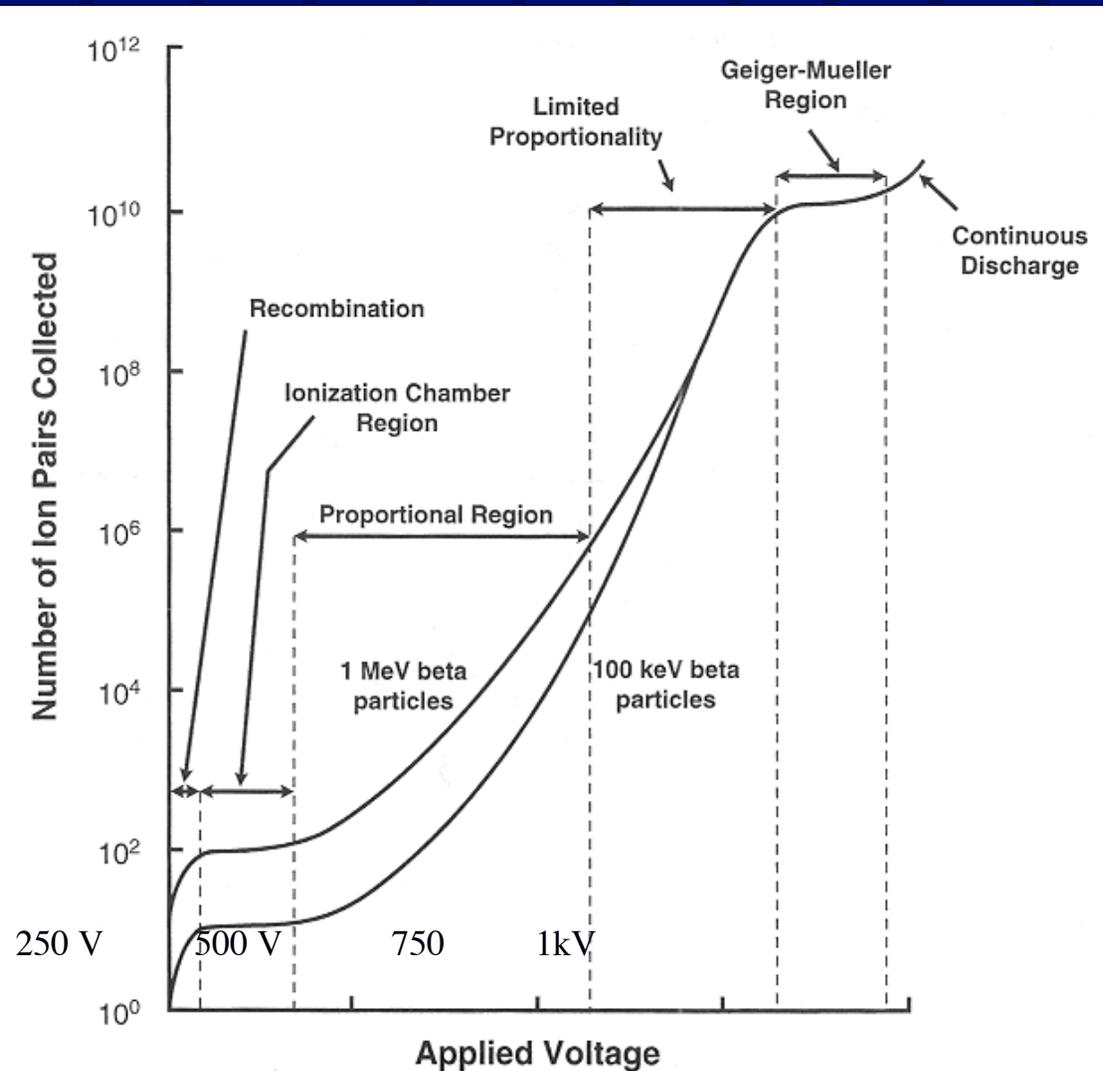
MultiWire Proportional Chambers

Charpak

An array of thin parallel anode wires (each acting as prop. counters): the field near the wires depends on their dimension. Wire spacing determines position resolution
Rule of thumb: diameter 1% of distance between wires. A mechanical problem is due to the wire repulsion so the tension of wire must overcome a minimum value that is a function of $(Vl/a)^2$
l = length of wires a = half gap anode-cathode
Typical set of parameters:
20 μm wire diameter 2 mm anode-wire distance
6 mm anode-cathode distance
Gas 80% Ar 20% isobutane with
quenching gas (methylal)
Cathode may be segmented or ground of connected wires in a perpendicular direction
so that the resolution is pads/sqrt(12)
sqrt(12) = σ of uniform dr
Can lead to resolutions of few tens μm

Hw7
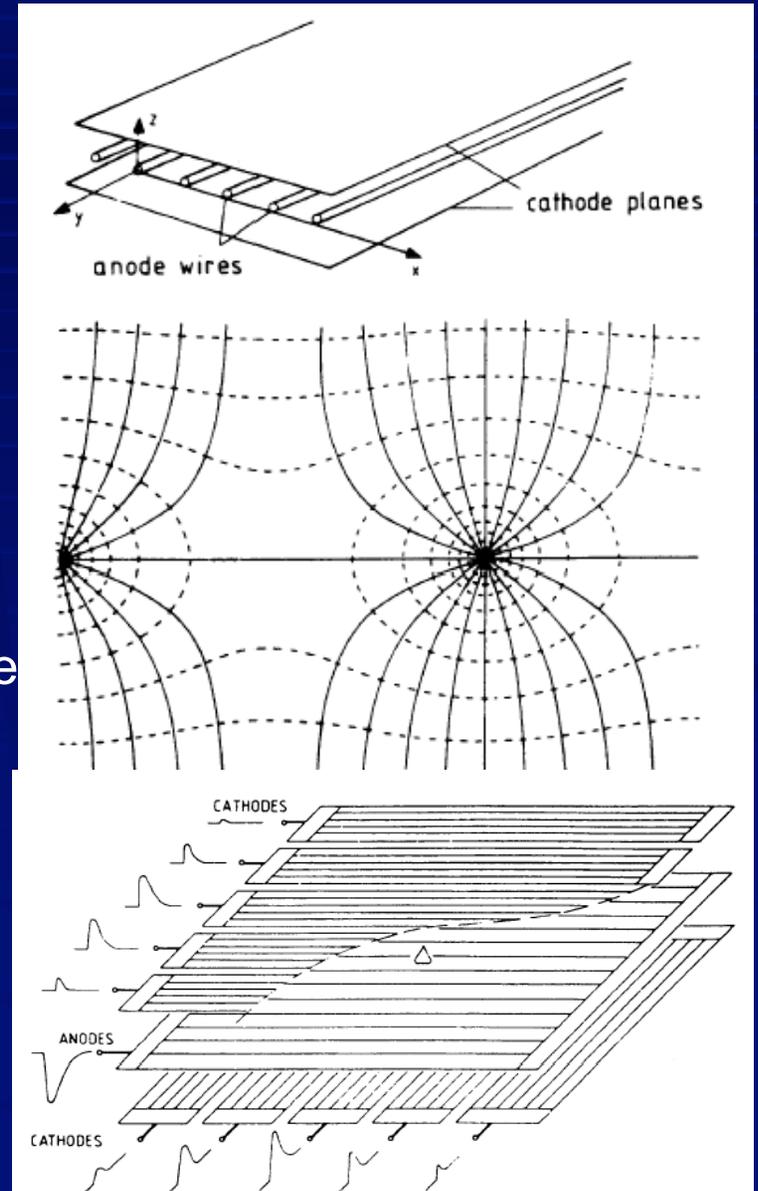
Planar Drift chambers

In systems where an external reference time is available, the time required for primary charges to reach the avalanche region can be used for position sensing Electrons formed along the track drift towards the central wire. The first electron to reach the high-field region initiates the avalanche which is used to derive the timing pulse. The drift path of electrons is
The drift velocity is constant if electric field strength is constant along the drift path.

$$z = \int_{t_0}^{t_1} v_D(t)dt = v_D \Delta t$$

Achievable position resolution
(typically better than MWPC) is ~200 μm
for a drift velocity of 500 mm/μs and
a time resolution of 4 ns.

Drift Chamber

anode

E

Proton

stop

start

Scintillation Counter
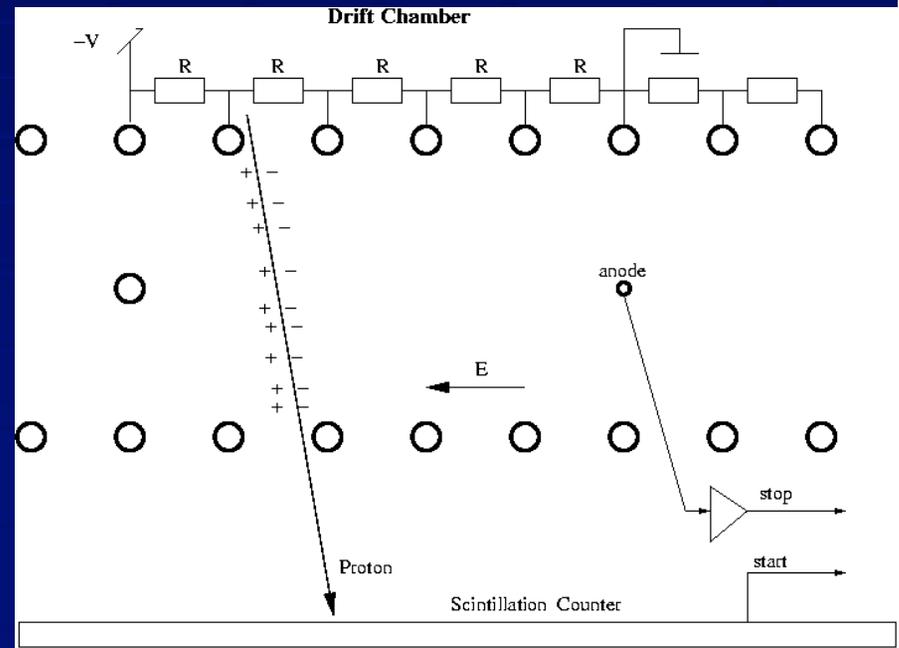
Drift Distance = Drift Velocity * Time

Drift chambers

In a conventional proportional chamber with parallel anode wires between 2 cathode planes it is not possible to have a constant E because of the zero field region between the 2 anode wires. A possibility is introducing a field wire at a potential -HV1 between 2 anode wires at +HV2.

Due to the electrical potentials applied to the wires the electrons drift to the sense wire.The difference between the time at which the signal is measured and the time when the particle traversed the cell (measured by other detectors) is used to reconstruct the impact point of the particle in the chambers midplane.
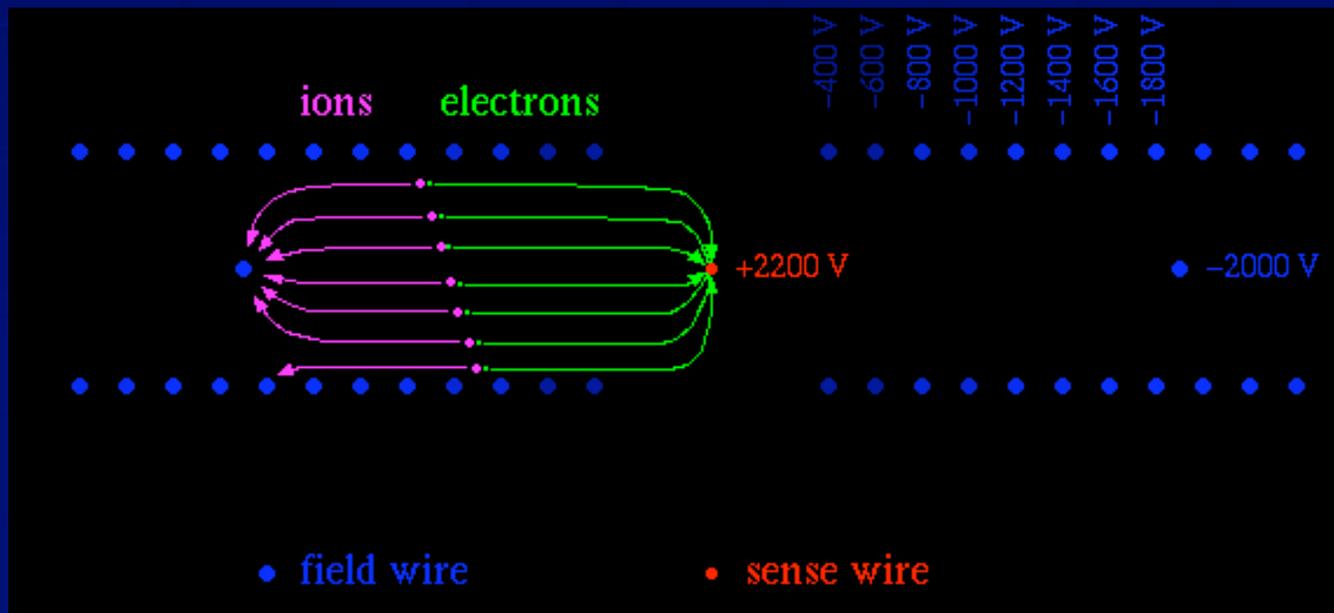
$$z = \int_{t_0}^{t_1} v_D(t)\,dt = v_D \Delta t$$

Jet Drift chambers

Cylindrical drift chamber are often used in colliding machines where detectors around the interaction point are inside solenoidal magnetic fields (the field is in the beam direction). The anode wires are along the B field direction and so E is radially directed. Electrons are deflected by the Lorentz force but since drift space is small (10 mm) the error is small too.

In jet drift chambers the number of measured points along a radial track is considerably increased thanks to the geometrical structure: drift cells form sectors of the cylinder with staggered sense wires to resolve left-right ambiguities

Geiger Mueller

if the electric field strength is higher than what used in the proportional mode, the number of UV photons formed in the avalanche process increase considerably. They create photoelectrons in the whole gas and in the walls of the counter. The avalanche spreads over the whole counter and leads to a complete discharge. The number of electrons produced is independent of the applied voltage and independent on the primary ionization. When the Geiger-Mueller region sets in (gas amplification of about 10^8-10^{10}) the number of ion pairs liberated then becomes equal for particles inducing different primary ionization, namely electrons or α. These counters do not allow to discriminate incoming particles.