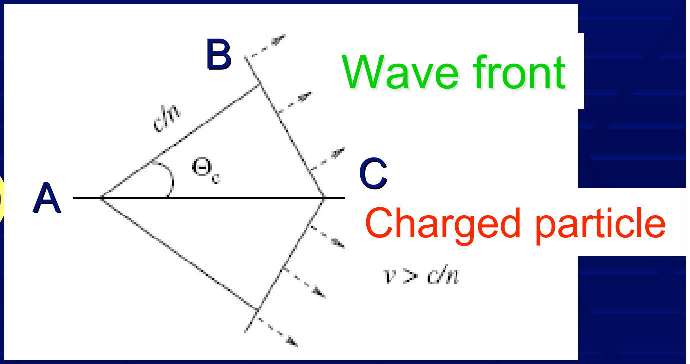
The Cherenkov effect

A charged particle traveling in a dielectric medium with n>1 radiates **Cherenkov radiation** if its velocity is larger than the phase velocity of light **$v>c/n$ or $\beta > 1/n$ (threshold)**



The emission is due to an asymmetric polarization of the medium in front and at the rear of the particle, giving rise to a varying electric dipole momentum.

Some of the particle energy is converted into light. A coherent wave front is generated moving at velocity v at an angle Θ_c

If the media is transparent the Cherenkov light can be detected.

If the particle is ultra-relativistic $\beta \sim 1$ Θ_c = const and has max value

In water $\Theta_c = 43^\circ$, $\cos\theta_c = \frac{AB}{AC} = \frac{\frac{c}{n}t}{\beta ct} = \frac{1}{\beta n}$

The Cherenkov effect

The intensity of the Cherenkov radiation (number of photons per unit length of particle path and per unit of wave length)

$$\frac{d^2N}{dxd\lambda} = \frac{4\pi^2 z^2 e^2}{hc\lambda^2}\left(1 - \frac{1}{n^2\beta^2}\right) = \frac{2\pi z^2}{\lambda^2}\alpha \sin^2\Theta_C$$

$$\alpha = \frac{2\pi e^2}{hc}$$

Number of photons/L and radiation Wavelength depends on charge and velocity of particle
Since the intensity is proportional to $1/\lambda^2$ short wavelengths dominate

Using light detectors (photomultipliers) sensitive in 400-700 nm for an ideally 100% efficient detector in the visible

$$\frac{dN_\gamma}{dx} = 491$$

$$\frac{dN_\gamma}{dx} = \int_{\lambda_1}^{\lambda_2} d\lambda \frac{d^2N_\gamma}{dxd\lambda} = 2\pi z^2 \alpha \sin^2\Theta_C \int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\lambda^2} = 2\pi z^2 \alpha \sin^2\Theta_C \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = 490\, z^2 \sin\Theta_c \quad photons/cm$$

$$\frac{d^2N}{dxdE} = \frac{d^2N}{dxd\lambda}\frac{d\lambda}{dE} = \frac{\lambda^2}{2\pi\cancel{h}c}\frac{d^2N}{dxd\lambda}$$

$$E = h\nu = \frac{hc}{\lambda} = \frac{2\pi\cancel{h}c}{\lambda}$$

$$\frac{d^2N}{dEdx} = \frac{\alpha z^2}{\hbar c}\sin^2\theta_c = \frac{\alpha^2 z^2}{r_e m_e c^2}\left(1 - \frac{1}{\beta^2 n^2(E)}\right)$$
$$\approx 370 \sin^2\theta_c(E)\ \mathrm{eV^{-1}cm^{-1}} \qquad (z = 1)\ ,$$

Energy loss is about 10^4 less than 2 MeV/cm in water from ionization but directional effect

Transition Radiation

Theory: Ginzburg, Frank 1946

Radiation is emitted when a fast charged particle crosses the boundary between two media with different indices of refraction (even below Cherenkov threshold)

Radiation is due to a coherent superposition of radiation fields generated by polarization of the medium (given a charge moving towards the 2 media interface it can be considered together with its mirror charge an electric dipole whose field strength varies with time. The time dependent dipole field causes the emission of electromagnetic radiation). Coherence is assured in a small region whose extension is called coherent length or formation zone. An observable amount of X-rays can be emitted when a particle with $\gamma >> 1$ crosses the boundary of a macroscopically thick medium. The number of emitted photons can be enhanced by radiators consisting of several boundaries.

Transition radiation

When a particle with charge ze crosses the boundary between vacuum and a medium with plasma frequency ω_p the energy radiated is

$$I = \alpha z^2 \gamma \hbar \omega_p / 3$$

Where the plasma frequency of the gas of electrons of the material is

$$\nu_p = \frac{\omega_p}{2\pi} = \sqrt{\frac{N_e e^2}{\pi m_e}}$$

And the emitted photon energy is

$$h\nu_p = h\sqrt{\frac{Z\rho N_A e^2}{\pi m_e A}} = \sqrt{4\pi N_A r_e \hbar^2 c^2}\sqrt{\frac{Z\rho}{A}} \approx 28.8 eV \sqrt{\frac{Z\rho}{A}}$$

Photons are emitted in a cone of half-aperture $\theta \sim 1/\gamma$ and the emission is peaked in the forward direction

The radiation yield drops fast for $\omega > \gamma\omega_p$ and diverges logaritmically at low energies. For a particle with $\gamma = 1000$ the radiated photons are in soft X-ray range between 2-20 keV. The number of emitted photons above an energy is $N_\gamma \propto z^2 \alpha$. The radiation probability is of order of $\alpha = 1/137$ hence the necessity to have many boundaries

Transition Radiation

When the interface is between 2 media of plasma frequencies $\omega_{p1,2}$ the energy radiated by the particle of charge ze at the boundary per unit solid angle and unit frequency is

$$\frac{d^2W}{d\nu d\Omega} \approx \frac{z^2 h\alpha}{\pi}\theta^2\left(\frac{1}{\gamma^{-2}+\theta^2+\Upsilon_1^2}-\frac{1}{\gamma^{-2}+\theta^2+\Upsilon_2^2}\right)^2$$

$$\Upsilon_{1,2} = \frac{\omega_{p,1,2}}{\omega}$$

Where θ is the angle between the particle and the emitted photon.

Three regions can be identified as a function of γ:

1) $\gamma \ll 1/Y_1 \Rightarrow$ low yield $$\frac{d^2W}{d(h\nu)d\Omega} \approx \frac{z^2\alpha}{6\pi}(\gamma Y_1)^4$$

2) $1/Y_1 \ll \gamma \ll 1/Y_2 \Rightarrow$ log increase with γ (used for PID) $$\frac{d^2W}{d(h\nu)d\Omega} \approx \frac{2z^2\alpha}{\pi}[\ln(\gamma Y_1)-1]$$

3) $\gamma \gg 1/Y_2 \Rightarrow$ saturation (yield is constant)

The total emitted energy in the forward direction is proportional to $z^2\gamma$

$$W = \int_0^\infty \frac{dW}{d(h\nu)}d(h\nu) \approx z^2\gamma\frac{\alpha h}{3}\frac{(\nu_{p,1}-\nu_{p,2})^2}{\nu_{p,1}+\nu_{p,2}}$$

Emission in radiators and TR detectors

For a foil of thickness l_1:
Interference produces
a threshold effect in γ

$$\left[\frac{d^2W}{d(h\nu)d\Omega}\right]_{foil} = 4\sin^2\left(\frac{\varphi_1}{2}\right)\frac{d^2W}{d(h\nu)d\Omega}$$

Interference term

$$\varphi_1 \approx \frac{(\gamma^{-2}+Y_1^2)\omega l_1}{2c}$$

Formation zone

For foil thickness << $Z_1(\omega) = \dfrac{2c}{(\gamma^{-2}+Y_1^2)\omega}$ the yield is strongly suppressed

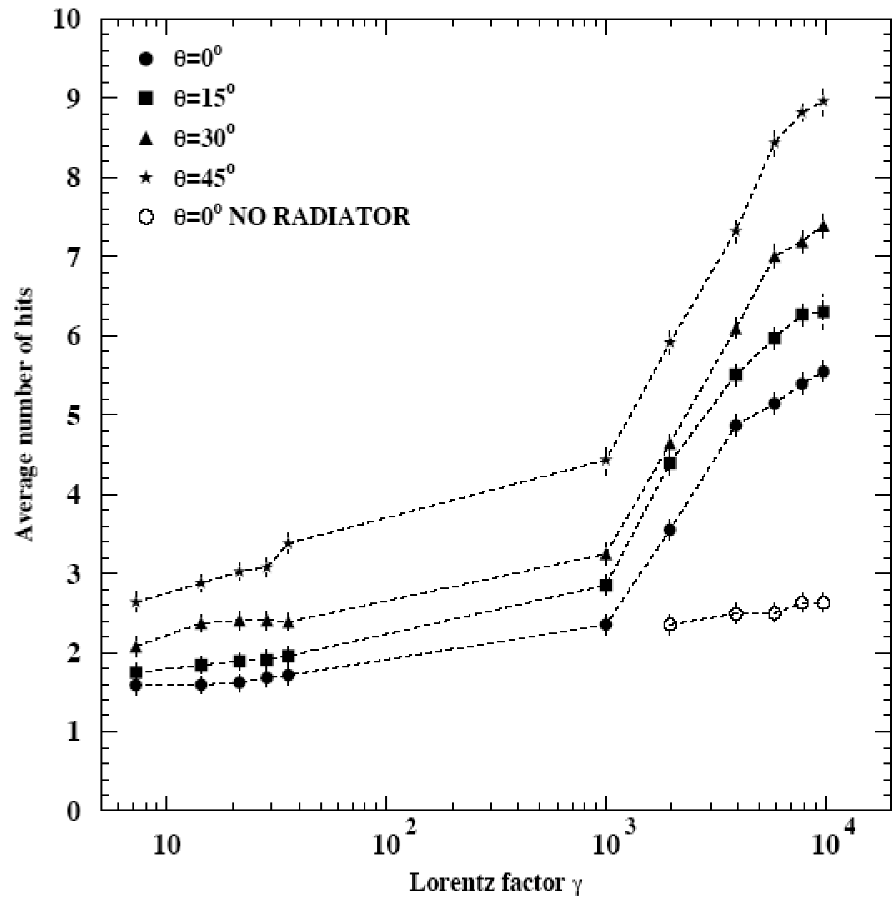
For large ω in order to have radiation emission

$$l_1 \geq Z_1(\omega) \approx \frac{2c\gamma^2}{\omega}$$

When many thin layers of material are joined together the number of photons can be increased and the limiting factor is reabsorption in the radiator itself. In a TR detector, to minimize the photoelectric cross section ($\propto Z^5$) materials with small Z (eg Li) must be chosen while the γ detectors can be gas detectors with large Z (eg Xe). TR detectors are characterized by a threshold

An example: calibration of MACRO TRD

TRD exposed to a pion/electron beam with various crossing angles. Number of hits in proportional counters vs γ of incident particles. Below the threshold only ionization contributes, for $10^3 < \gamma < 10^4$ TR is the main contribution and the number of hits increases logaritmically with γ. For larger values it saturates



Suggested readings

Textbooks:

Konrad Kleinknecht, Detectors for Particle Radiation (2nd edition) Cap 1

C. Leroy and PG Rancoita, Principles of Radiation Interaction in Matter and detection, World Scientific, 2004 (Cap 2)

M.S. Longair, High Energy Astrophysics, Third ed., Vol 1, Particles, photons and their interactions (Cap 2-4)

Online:

http://pdg.lbl.gov/2002/passagerpp.pdf

R.K. Bock & A. Vasilescu - The Particle Detector BriefBook, Springer 1998

http://physics.web.cern.ch/Physics/ParticleDetector/BriefBook/

http://www.shef.ac.uk/physics/teaching/phy311/#books

http://www-physics.lbl.gov/%7Espieler/physics_198_notes_1999/index.html

http://besch2.physik.uni-siegen.de/%7Edepac/DePAC/DePAC_tutorial_database/grupen_istanbul/grupen_istanbul.html