## Energy

$$\text{Potential energy} = \frac{1}{2} m\omega^2 x^2$$

$$= \frac{1}{2} m\omega^2 e^{-\gamma t} \left( \frac{A e^{i\sqrt{4\omega_0^2-\gamma^2}\,t} + c.c.}{2} \right)^2$$

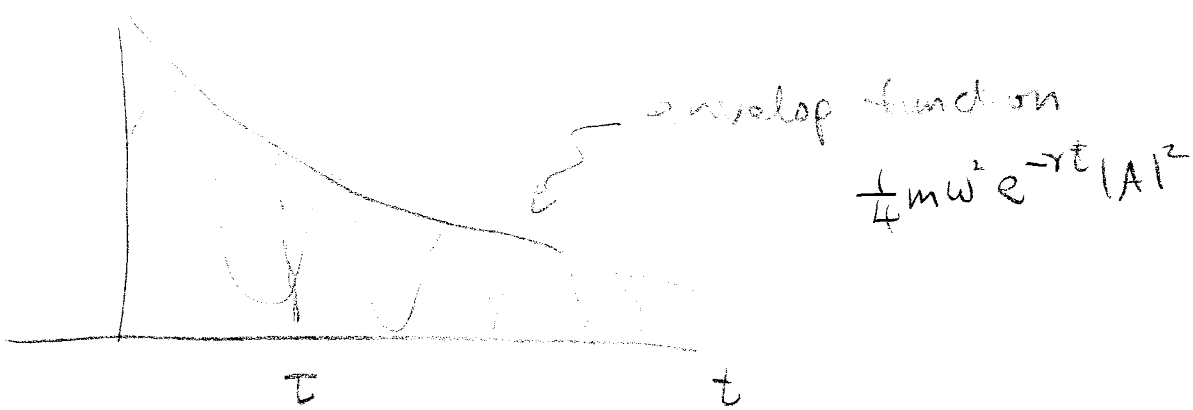
complex conjugate

$$= \frac{1}{2} m\omega^2 e^{-\gamma t} \frac{1}{4} \Big[ A^2 e^{2i\sqrt{4\omega_0^2-\gamma^2}\,t} + 2AA^* + A^{*2} e^{-2i\sqrt{4\omega_0^2-\gamma^2}\,t} \Big]$$

On average, $\langle A^2 e^{i2\sqrt{4\omega_0^2-\gamma^2}\,t} + A^{*2} e^{-i2\sqrt{4\omega_0^2-\gamma^2}\,t} \rangle$

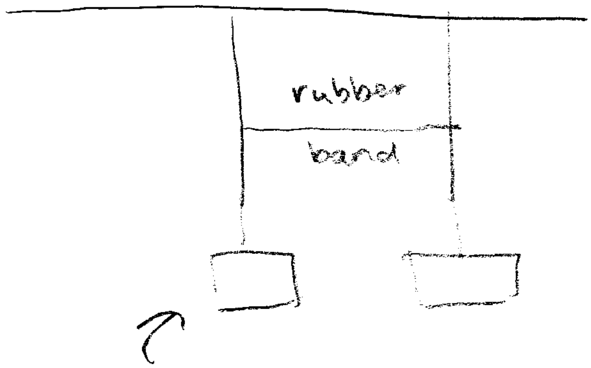
$$= 0$$

$$\Rightarrow \langle \text{Potential energy} \rangle = \frac{1}{4} m\omega^2 e^{-\gamma t} |A|^2$$




Time constant $\boxed{\tau = \frac{1}{\gamma}}$

# Coupled Oscillators (Demo)




$$\ddot{X}_1 = -\omega_0^2 X_1 + \beta X_2$$

$$\ddot{X}_2 = -\omega_0^2 X_2 + \beta X_1$$

Try: $X_1 = A_1 e^{i\omega t}$

$X_2 = A_2 e^{i\omega t}$

$A_1, A_2$ fixed by initial conditions

$$\Rightarrow \quad -\omega^2 A_1 = -\omega_0^2 A_1 + \beta A_2$$

$$-\omega^2 A_2 = -\omega_0^2 A_2 + \beta A_1$$

$$\Rightarrow \quad A_2 = \frac{(\omega_0^2 - \omega^2) A_1}{\beta} = \frac{\beta A_1}{\omega_0^2 - \omega^2}$$

$$\Rightarrow \quad \omega^2 = \omega_0^2 \mp \beta$$

More later in Sophomore / junior year

# Driven Oscillator (Demo)

$$m\ddot{x} = -m\omega_0^2 x + F\cos\omega t$$

driving frequency

$$\ddot{x} = -\omega_0^2 x + \frac{F}{m}\left(\frac{e^{i\omega t}+e^{-i\omega t}}{2}\right) \qquad (*)$$

First, solve

$$\ddot{x} = -\omega_0^2 x + \frac{F}{2m}e^{i\omega t} \qquad (1)$$

then $\omega \to -\omega$ of solution satisfies

$$\ddot{x} = -\omega_0^2 x + \frac{F}{2m}e^{-i\omega t} \qquad (2)$$

Adding the solutions give a solution to (*)

To solve (1), try $x = Ae^{i\omega t}$

$$-\omega^2 A e^{i\omega t} = -\omega_0^2 A e^{i\omega t} + \frac{F}{2m}e^{i\omega t}$$

$$A = \frac{F}{2m(\omega_0^2-\omega^2)}$$

Putting in damping

$$\ddot{X} = -\omega_0^2 + \frac{F}{m}\left(\frac{e^{i\omega t} + e^{-i\omega t}}{2}\right) - \gamma \dot{X}$$

Again solve $e^{i\omega t}$ part,

Try $X = A e^{i\omega t}$

$$\Rightarrow -\omega^2 A e^{i\omega t} = -\omega_0^2 A e^{i\omega t} + \frac{F}{2m} e^{i\omega t} - i\omega\gamma A e^{i\omega t}$$

$$\Rightarrow \quad A = \frac{F/2m}{\omega_0^2 - \omega^2 + i\omega\gamma}$$

Hence, $X(t) = \dfrac{F/2m}{\omega_0^2 - \omega^2 + i\omega\gamma} e^{i\omega t} + \text{c.c.}$

For $\gamma = 0$

$$X(t) = \frac{F/2m}{\omega_0^2 - \omega^2}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

$$= \frac{F/m}{\omega_0^2 - \omega^2} \cos\omega t$$