PHY 248 Lecture 7

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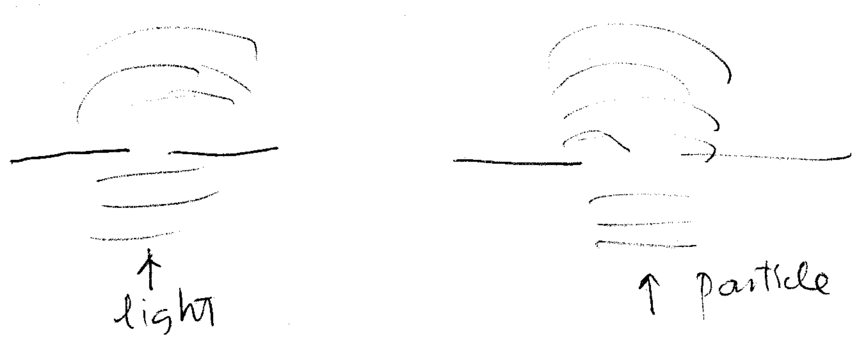
Waves

- oscillations (e.g. pendulum, spring-mass system)
- waves – space & time dependent oscillations)
- light waves – interference & diffraction

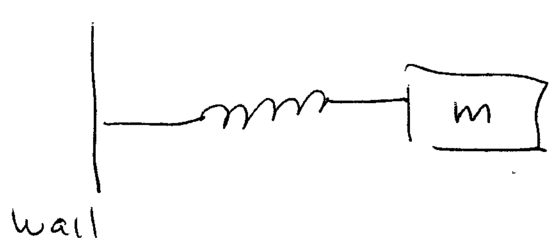
→ lead us to <u>Quantum Theory</u>: particles are waves!

In fact, the uncertainty principle can be understood in terms of diffraction



$\Delta p \Delta x \geq \frac{\hbar}{2}$

Simplest example of oscillations



$F = -kx$

↑ displacement of spring

Hooke's law

$$ma = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \equiv -\omega^2 x \qquad \omega \equiv \sqrt{\frac{k}{m}} = \text{angular freq}$$

To solve this equation, let's recall that sine and cosine have the property that their 2nd derivatives are proportional to themselves

$$\frac{d^2}{dx^2}\sin x = -\sin x$$

$$\frac{d^2}{dx^2}\cos x = -\cos x$$

<u>Guess</u> : $x(t) = A\cos(\omega t+\delta)$

A = amplitude (arbitrary set by Initial conditions)

δ = phase

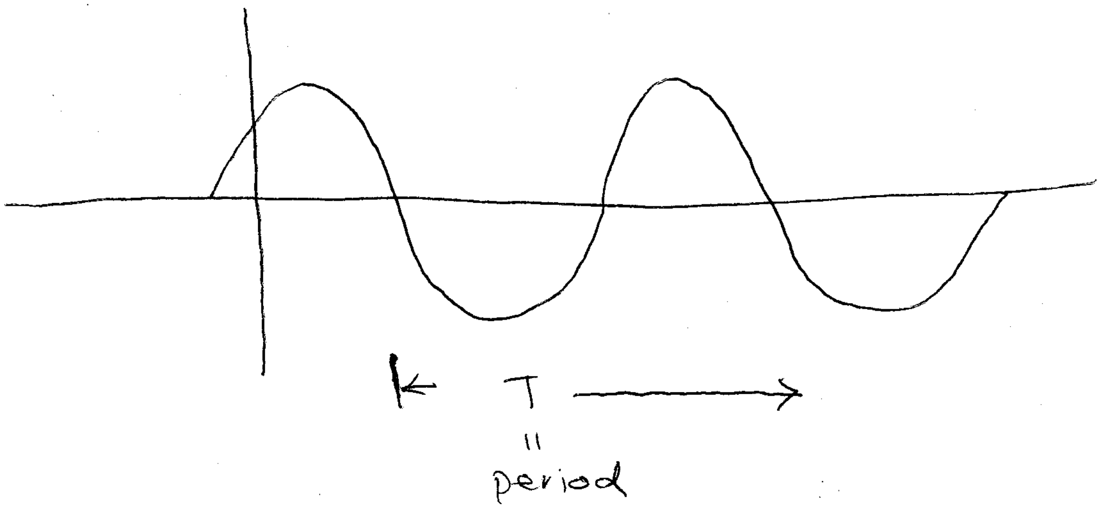
Note that by including δ, we cover both the cases of sine & cosine

<u>Check</u> :

$$\frac{dx}{dt} = -A\omega\sin(\omega t+\delta)$$

$$\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t+\delta) = -\omega^2 A \quad ✓$$

Picture



$$\omega T = 2\pi \qquad \text{cosine repeats}$$

$$\omega = \frac{2\pi}{T}$$

Frequency $= f = \frac{1}{T} =$ # of cycles per unit time

$$= \frac{\omega}{2\pi}$$

Initial conditions:

say $x(t=0) = x_0$

$\dot{x}(t=0) = 0$

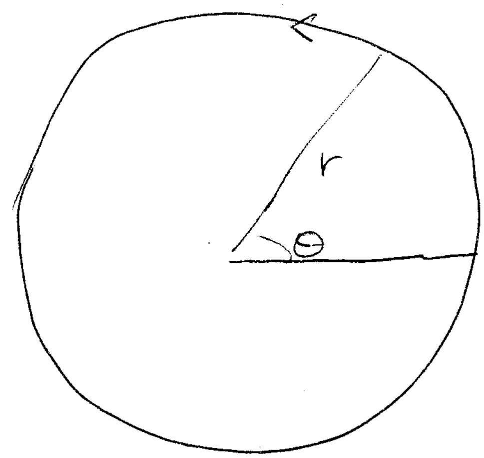
$\Rightarrow \quad A\cos\delta = x_0$

$-A\omega\sin\delta = 0$

$\Rightarrow \quad \delta = 0$ (actually integer multiple of π)

$A = x_0$

Example: Circular motion



$\theta = \omega t$ constant speed

$x = r\cos\theta = r\cos\omega t$

$y = r\sin\theta = r\sin\omega t$

Solution is exactly the type $x = A\cos(\omega t + \delta)$ discussed before.

Energy of an oscillatory system

$$E = K + U$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$= \frac{1}{2}m(-\omega A\sin(\omega t+\delta))^2 + \frac{1}{2}(m\omega^2)[A\cos(\omega t+\delta)]^2$$

$$= \frac{1}{2}m\omega^2 A^2 \left[\sin^2(\omega t+\delta) + \cos^2(\omega t+\delta)\right]$$

kinetic potential

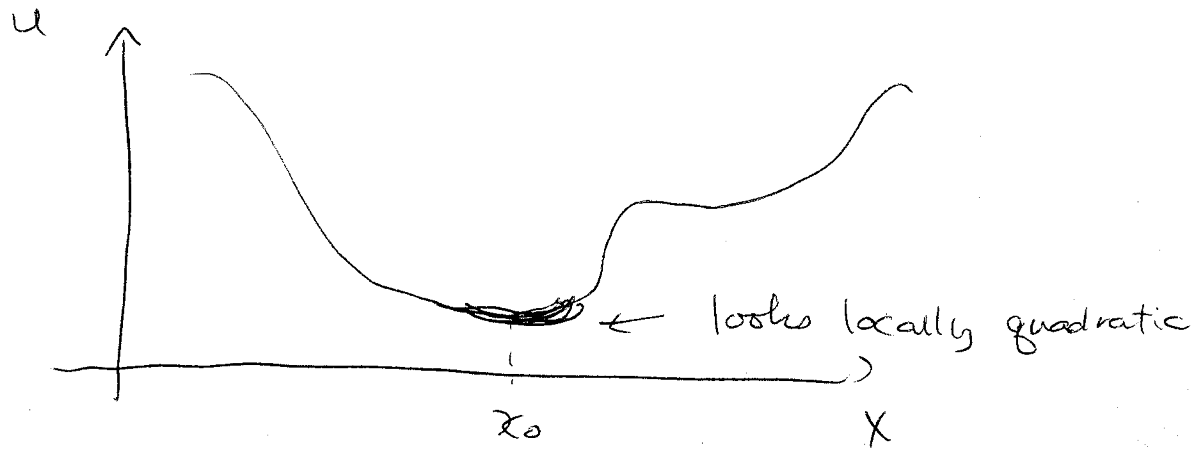
trading but holding total fixed

$$= \frac{1}{2}m\omega^2 A^2 = \text{fixed}$$

amplitude^2

Simple Harmonic Oscillation (SHO)

Very common in physics.



$$F = -\frac{du}{dx}$$

$$m\frac{d^2x}{dt^2} = -\frac{du}{dx}$$

Expand around minimum x_0

$$u(x) = u(x_0) + \underbrace{u'(x_0)}_{0\text{ if } x_0 \text{ is a minimum}}(x-x_0) + \frac{1}{2}u''(x_0)(x-x_0)^2 + \cdots$$

$$= u(x_0) + \frac{u''(x_0)}{2}(x-x_0)^2$$

$$F = -\frac{du}{dx} = -u''(x_0)(x-x_0)$$ just like Hooke's law

Small oscillations about x_0

$$k = \left.\frac{d^2U}{dx^2}\right|_{x=x_0}$$

$$\omega = \sqrt{\frac{k}{m}}$$

To increase ω, we can increase the curvature of U

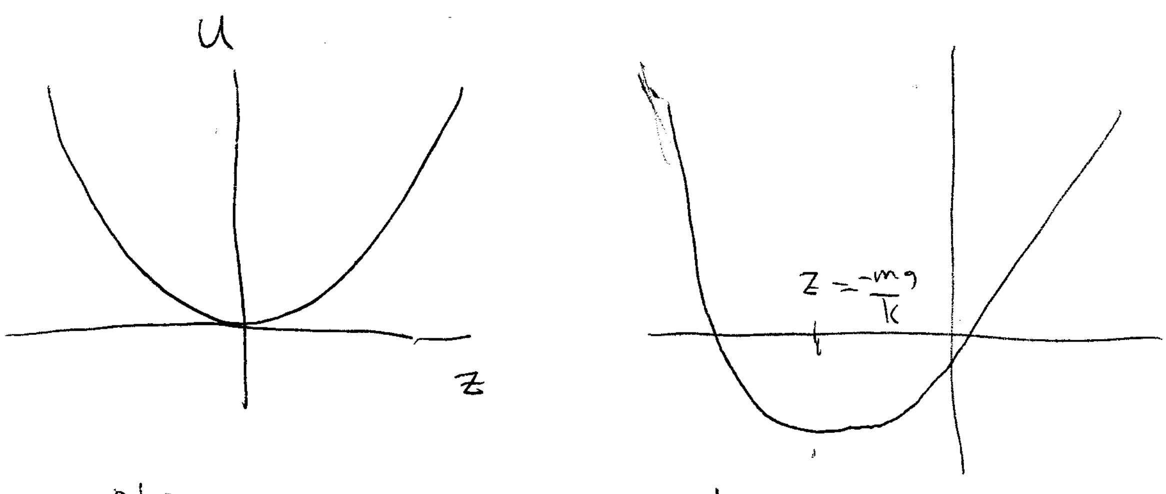
Demo?

<u>Example</u> vertical spring

$$U = \frac{1}{2}kz^2 + mgz$$

Equilibrium $\frac{dU}{dz} = 0 = kz + mg$

$$\Rightarrow z = -\frac{mg}{k}$$



No gravity w/ Gravity