PHY 248 Lecture 4 Gary Shiu 1/2

General Relativity (T & L Ch 2)

Newton's law of Gravitation $\vec{F} = -\frac{GMm}{R^2}\hat{r}$

Introduce gravitational field $\vec{g} = \underbrace{-\frac{GM}{R^2}}_{\text{properties of source}}\hat{r}$

How do objects far away know about each other?

Action at a distance? violates relativity

Answer: field transmit energy

$\therefore$ field stores energy

But special relativity $\Rightarrow$ Energy $\simeq$ mass

$\Rightarrow$ gravitational field generates further gravitational field ..... keep looping forever!

$$\vec{F} = \frac{d\vec{p}}{dt}$$

what should we put here?

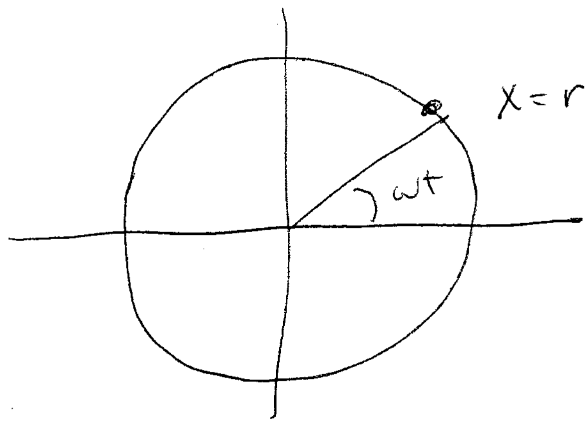
General relativity : incorporates gravity

Equivalence Principle

Gravity is a fictitious force

What does that mean by fictitious force?

Analogous example : circular motion



$x = r\cos\omega t \quad y = r\sin\omega t$

$\boxed{\omega = \frac{v}{r}}$

→ acceleration $\quad \vec{F} = m\vec{a} = -\frac{mv^2}{r}\hat{r}$

Go to a rotating frame $\quad x' = x - r\cos\omega t$

$y' = y - r\sin\omega t$

then $\quad x' = y' = 0 \quad$ no force at all

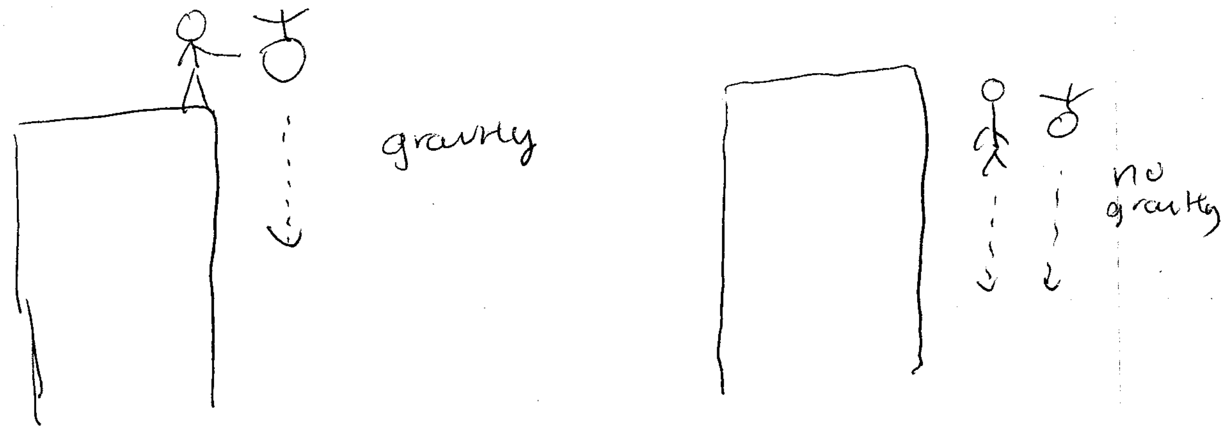
Suppose we view a stationary particle in rotating frame

$$\left.\begin{aligned} x' &= -r\cos\omega t \\ y' &= -r\sin\omega t \end{aligned}\right\} \text{ rotating hence } \vec{F} = -\frac{mv^2}{r}\hat{r}$$

Acceleration can be made to disappear/appear by going to a proper coordinate system

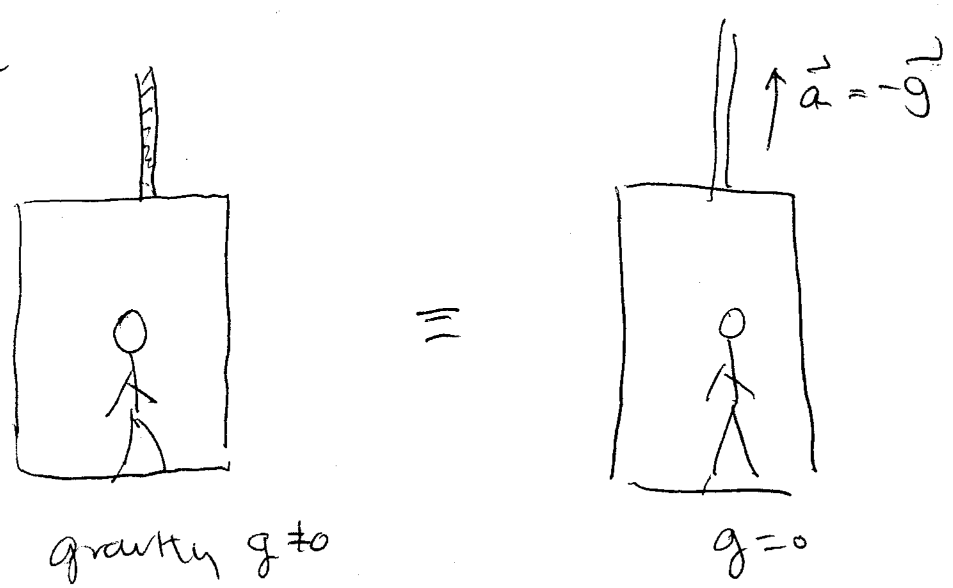
<u>Principle of Equivalence</u> says that gravity is such a force!

<u>Thought Expt</u>



<u>Einstein</u> : happiest thought of his life

Likewise



Important: $m_G = m_I \Rightarrow \vec{g}$ = same for all objects

Einstein elevated this fact to a <u>principle</u>:
this equivalence applies to <u>all</u> physics (not only gravity but EM etc)

More clearly stated: Equivalence Principle

$\boxed{\text{Physics in a free-falling frame with gravity}} \equiv \boxed{\text{Physics in an inertial frame without gravity}}$

$\boxed{\text{Physics in a non-accelerating frame with gravity } \vec{g}} \equiv \boxed{\text{Physics in a frame without gravity but } \vec{a} = -\vec{g}}$

Effects of gravity

$$\psi = \frac{E_{grav}}{E_{rest}} = \frac{m_G \Phi}{m_I c^2} = \frac{\Phi}{c^2}$$

$\psi \ll 1$ typically, e.g. on earth surface
(Newtonian limit)

$$\Phi \sim gh \sim (10\, m/s^2)(10\, m) \Rightarrow \frac{\Phi}{c^2} \sim 10^{-15} \text{ tiny}$$

(gravity is weak)

When $\psi \sim 1$ GR is important

Typically $\Phi \sim \frac{GM}{R}$, so $\psi \sim 1$ when

e.g. Black hole: compact object . $\frac{M}{R}$ is large
Cosmology: extremely massive

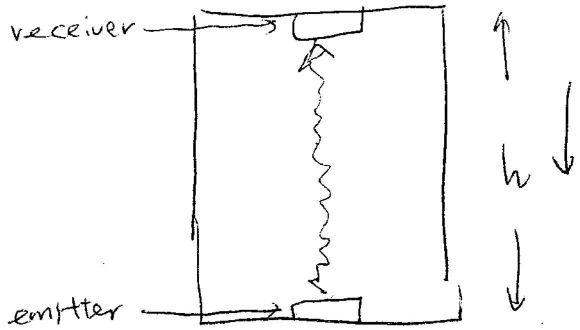
The Equivalence Principle (EP) motivates Einstein to formulate a <u>geometric</u> theory of gravity.

<u>EP Implications:</u> - bending of light

- gravitational redshift

Will discuss these ~~o~~ results from EP (before we formulate GR) but keep in mind that the <u>precise</u> quantitative answers (e.g. angle light is bent) would get further modified when the ~~###~~ proper framework of GR is developed.

Gravitational Redshift



Free falling elevator

EP $\Rightarrow$ inertial frame

no physical effects associated with gravity

- Free-falling frame

$$(\Delta\omega)_{ff} = (\omega_{rec} - \omega_{em})_{ff} = 0$$

- From the pt of view of us who see the elevator falling. $\Rightarrow$ there is gravity and the elevator is accelerating.

Time to reach receiver $\Delta t = \frac{h}{c}$ (distance elevator fell is negligible)

velocity of receiver after Δt is

$$\Delta u = g\,\Delta t = \frac{gh}{c}$$

Doppler's effect $\Rightarrow$ blue shift $f = f_0\sqrt{\frac{1+\beta}{1-\beta}}$

$$\left(\frac{\Delta\omega}{\omega}\right)_{Doppler} \simeq \beta = \frac{\Delta u}{c}$$ for small β

EP $\Rightarrow$ physics same in this reference frame

$$\Delta\omega = \omega_{rec} - \omega_{em} = 0$$

If not, we can measure the frequency (color) of received light to tell whether we are accelerating or gravity.

How can $\Delta\omega = 0$ be possible?

Ans: If gravity produces an opposite effect, i.e. light redshifted by gravity

$$\left(\frac{\Delta\omega}{\omega}\right)_{gravity} = -\frac{\Delta u}{c}$$

just the right amount to cancel Doppler blueshift

$$\Rightarrow \left(\frac{\Delta\omega}{\omega}\right)_{gravity} = -\frac{gh}{c^2}$$

Just like the gravitational field $\vec{g} = \frac{\vec{F}}{m}$

gravitational potential $\Phi = \frac{U}{m}$ (potential energy, not velocity)

$$= \frac{mgh}{m}$$

$$= gh$$

$$\left(\frac{\Delta\omega}{\omega}\right)_{gravity} = -\frac{\Delta\Phi}{c^2}$$

$$\frac{\omega_{rec} - \omega_{em}}{\omega_{em}} = - \frac{\Phi_{rec} - \Phi_{em}}{c^2}$$

Since $\Phi_{rec} > \Phi_{em}$, $\Delta\omega < 0$ redshift

even though emitter & receiver are not in relative motion.

The gravitational redshift is very small

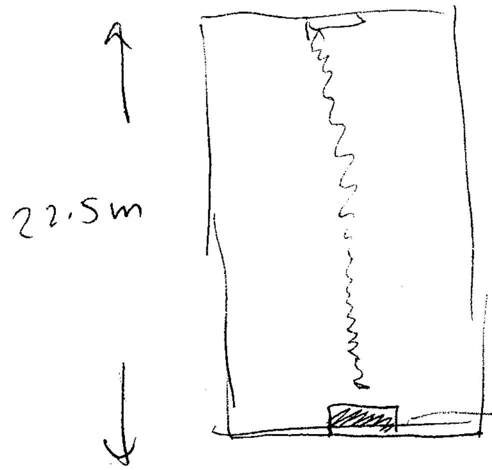
$$\frac{\Delta\omega}{\omega} = \frac{GM}{c^2 R} \qquad \text{spherical source}$$

$\Rightarrow$ noticeable if M is large and R is small
can easily be masked by thermal motion of emitter atoms

Surprisingly, gravitational redshift was first verified terrestrially by Pound and collaborators in the 1960s.

$$\left|\frac{\Delta\omega}{\omega}\right| = \left|\frac{gh}{c^2}\right| \sim O(10^{-15})$$

$h = 22.5$ m
= height of elevator shaft in Jefferson tower (Harvard physics dept)



$Fe^* \to Fe + \gamma$

Fe^{47} is chosen because we can fix the frequency of γ to high precision. Due to Mossbauer effect, the recoil of atoms can be ignored

Due to gravitational redshift,

γ can no longer be resonantly

absorbed at $h = 22.5$ m

To prove $\frac{\Delta \omega}{\omega} \sim O(10^{-15})$, move detector

slowly towards emitter so ordinary Doppler's

blueshift is just right to compensate grav. redshift

What is speed needed?

$$\frac{gh}{c^2} = \frac{u}{c}$$

$$u = \frac{gh}{c} = 7.35 \times 10^{-7} \text{ m/s}$$

takes a year ~~for~~ to cover the elevator shaft at same speed

recognize u can the speed an object attained in a time that light travels a distance h