PHYSICS 248 Lecture 19 Gary Shiu 3/12/07

Particle in a box : $E_n = \frac{\pi^2 \hbar^2}{2mL^2} n^2 \quad n = 1, 2, 3, \ldots$

Harmonic oscillator $E_n = (n + \frac{1}{2}) \hbar \omega \quad n = 0, 1, 2, \ldots$

More generally,

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

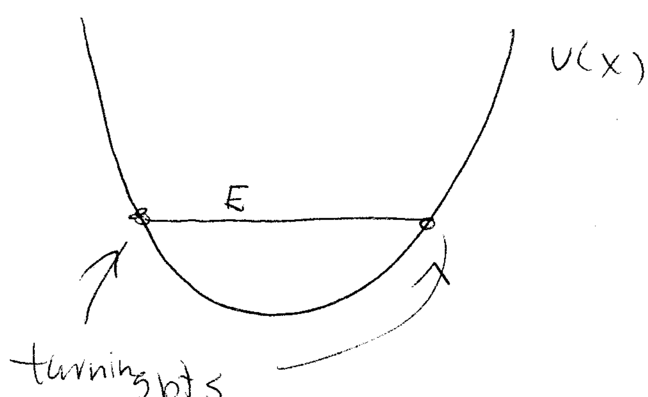
$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}(E - V)\psi$$

If $E > V(x)$ $\quad \frac{d^2\psi}{dx^2} = -k^2_{(x)}\psi$ where $k^2_{(x)} = \frac{2m}{\hbar^2}(E-V)$

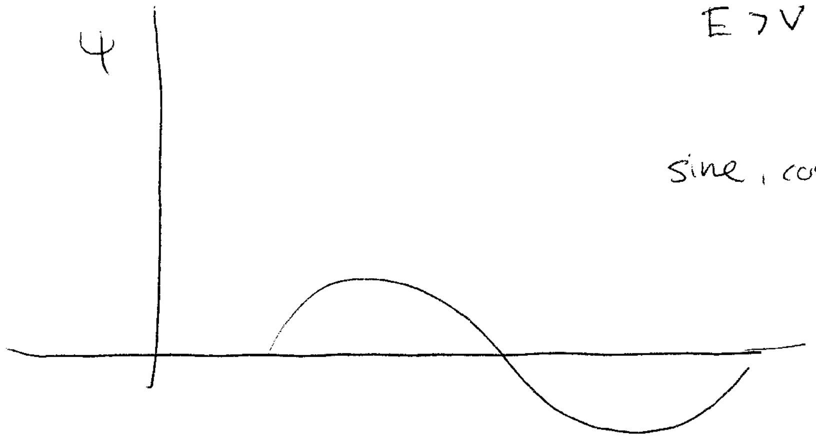
$\Rightarrow$ ψ looks like sine & cosine

$E < V(x)$ $\quad \frac{d^2\psi}{dx^2} = \alpha^2(x)\psi$ where $\alpha^2(x) = \frac{2m}{\hbar^2}(V-E)$

$$\Rightarrow \psi = e^{\pm \alpha x}$$



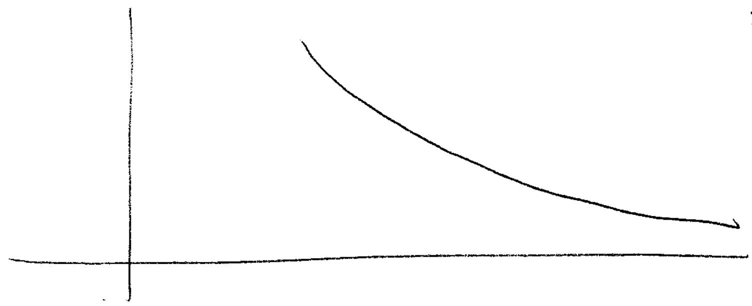
$E > V$



sine, cosine

concave towards x-axis

$E < V$



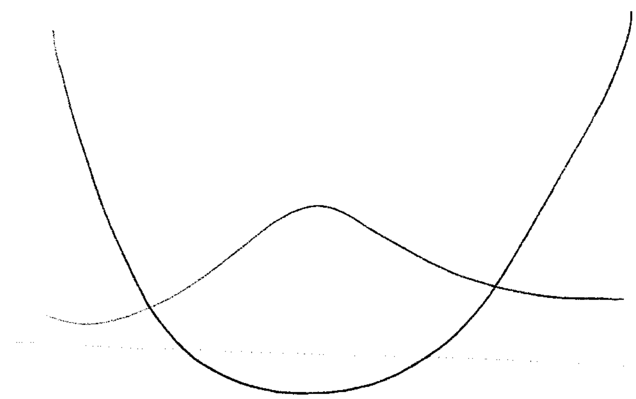
concave away from x-axis

These qualitative features together with

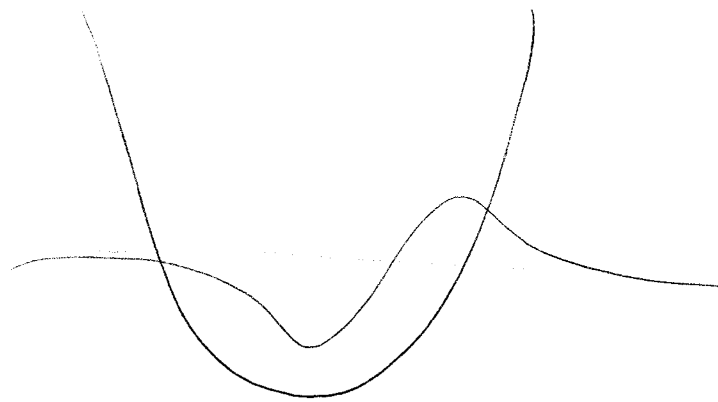
1. parity of wavefunctions
2. m-th excited state has m zero crossing
3. $P(x) \propto \frac{1}{v}$ classically

we can get a rather good idea of $\psi(x)$

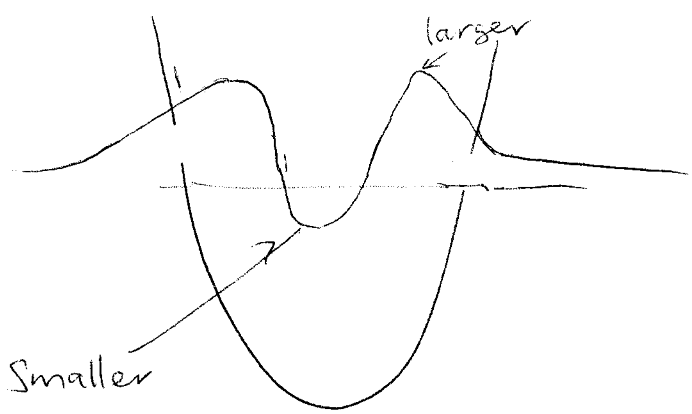
$n=0$



$n=1$



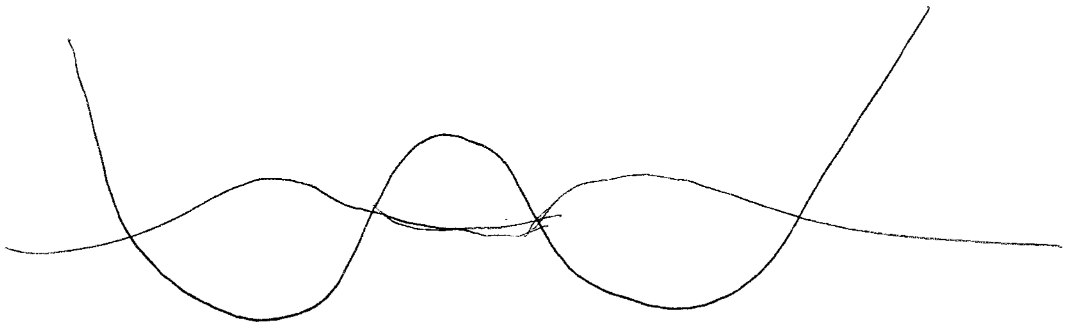
$n=2$



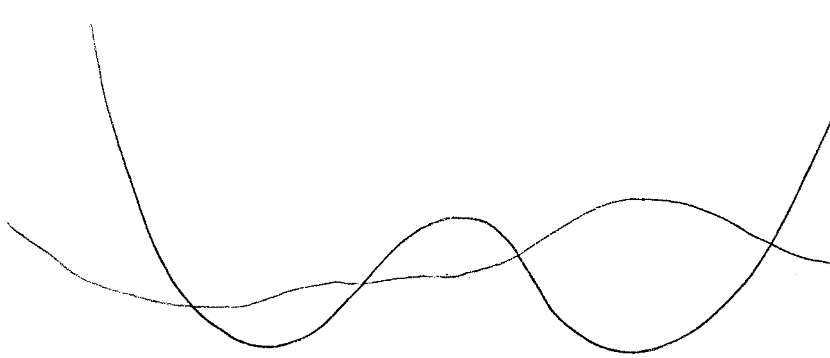
wavefunction should have smaller magnitude at the middle where the classical velocity is large.

Double well potential

$n=0$



$n=1$



Other simple generalizations

3D SHO :

$$V(x) = \frac{1}{2} K (x^2 + y^2 + z^2) = \frac{1}{2} m\omega^2 (x^2 + y^2 + z^2)$$

$$E = (n_1 + \tfrac{1}{2}) \hbar\omega + (n_2 + \tfrac{1}{2}) \hbar\omega + (n_3 + \tfrac{1}{2}) \hbar\omega$$

Energy levels are degenerate

Ground state $n_1 = n_2 = n_3 = 0$ $E_0 = \frac{3}{2} \hbar\omega$ non-degenerate

but

First excited state $n_1 = 1$, $n_2 = 0$, $n_3 = 0$
(or permutations)

$E_1 = \frac{5}{2} \hbar\omega$ 3-fold degenerate

More degeneracy at large E.