PHYSICS 248 Lecture 18 Gary Shiu

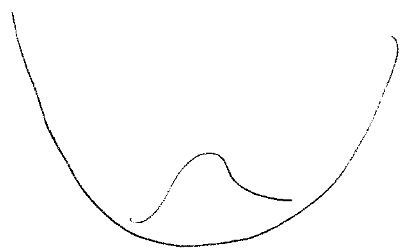
3/9/07

Instead of solving the Schrodinger equation, we will guess the answer

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2 x^2 \psi = E\psi$$

soln is not sine or cosine because of x^2 potential

Need a function that looks like



and generates x^2 term

Try $\psi(x) = A_0 e^{-qx^2}$ ← will determine this

↑ normalization

$$\psi' = A_0(-2qx)e^{-qx^2}$$

$$\psi'' = A_0(-2q)e^{-qx^2} + A_0(4q^2x^2)e^{-qx^2}$$

Substitute in Schrodinger equation

$$-\frac{\hbar^2}{2m}(-2q)A_0 e^{-qx^2} - \frac{\hbar^2}{2m}(4q^2x^2)A_0 e^{-qx^2}$$

$$+\frac{1}{2}m\omega^2x^2A_0e^{-qx^2} = E_0 A_0 e^{-qx^2}$$

Comparing coefficients of x^2 & 1

$$\begin{cases} -\dfrac{\hbar^2}{2m} 4q^2 = \dfrac{1}{2} m\omega^2 \\ \dfrac{\hbar^2}{2m}(2q) = E_0 \end{cases}$$

$$\Rightarrow \qquad q = \sqrt{\frac{m^2\omega^2}{4\hbar^2}} = \frac{m\omega}{2\hbar}$$

$$E_0 = \frac{\hbar^2}{m}\frac{m\omega}{2\hbar} = \frac{\hbar\omega}{2} \quad \checkmark$$

Hence $\psi(x) = A_0 e^{-\frac{m\omega}{2\hbar}x^2}$

Solves the schrodinger equation with

$$E = E_0 = \frac{\hbar\omega}{2}$$

Normalization

$$\int_{-\infty}^{\infty} |\psi_0(x)|^2 \, dx = 1$$

$$A_0^2 \int_{-\infty}^{\infty} e^{\frac{-m\omega x^2}{\hbar}} dx = 1 \qquad u = \sqrt{\frac{m\omega}{\hbar}}\, x$$

$$A_0^2 \sqrt{\frac{\hbar}{m\omega}} \underbrace{\int_{-\infty}^{\infty} e^{-u^2} du}_{\sqrt{\pi}} = 1 \Rightarrow \boxed{A_0 = \sqrt{\frac{m\omega}{\hbar\pi}}}$$

1st excited state

Try $\psi = A_1 x e^{-qx^2}$

qualitatively the same as infinite well potential

$$\psi'' = (4q^2x^3 - 6qx) A_1 e^{-qx^2}$$

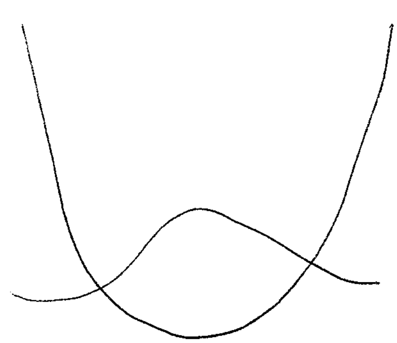
Schrodinger equation becomes

$$-\frac{\hbar^2}{2m}(4q^2x^3 - 6qx) A_1 e^{-qx^2} + \frac{1}{2} m\omega^2 x^3 A_1 e^{-qx^2}$$

$$= E x A_1 e^{-qx^2}$$

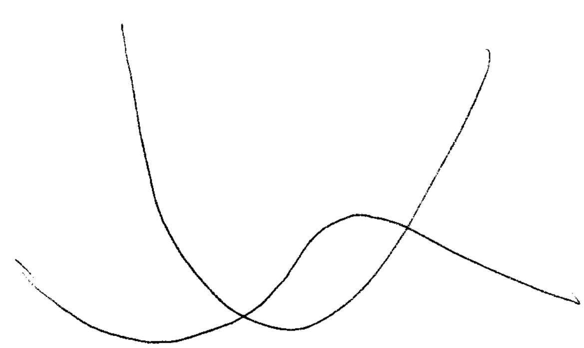
$\Rightarrow$ x^3 coefficient ; $-\frac{2\hbar^2}{m} q^2 + \frac{1}{2} m\omega^2 = 0 \Rightarrow q = \frac{m\omega}{2\hbar}$

x coefficient : $\frac{6\hbar^2 q}{2m} = E \Rightarrow E = \frac{3}{2}\hbar\omega$



$n=0$

even parity

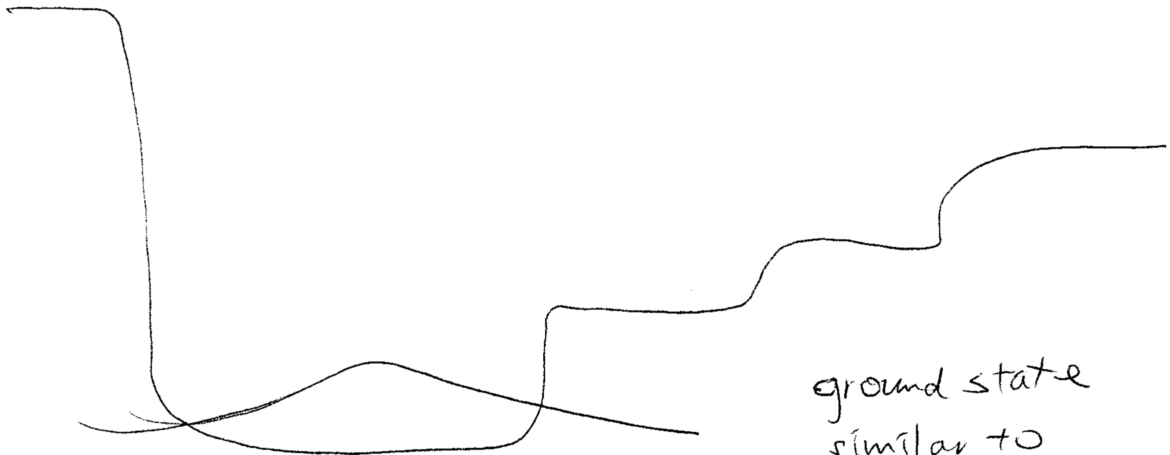


$n=1$

odd parity

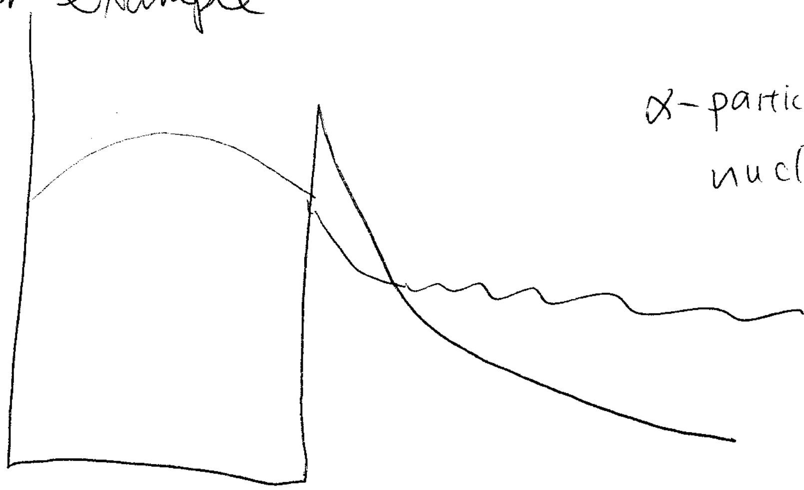
qualitatively the same as particle in a box

Consider



ground state similar to particle in a box

Another example



α-particle in gold nucleus