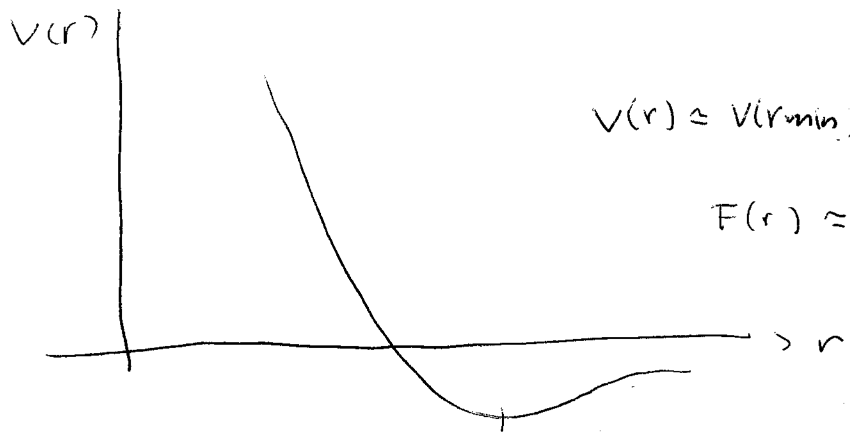
PHY 248 Lecture 17 Gary Shiu 3/7/07

Simple Harmonic Oscillator (SHO)

Classically $F = -kx$ or $m\ddot{x} = -kx$

Many applications; e.g. vibration in molecules



$V(r) \simeq V(r_{min}) + \frac{1}{2} V''(r_{min}) (r - r_{min})^2$

Taylor series

$F(r) \simeq -V''(r_{min}) r$

$k = V''(r_{min})$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{V''}{m}}$

Conservation of energy

$$\frac{1}{2} m v^2 + \frac{1}{2} m \omega^2 x^2 = E$$

$$v = \sqrt{\frac{2}{m}\left(E - \frac{1}{2} m\omega^2 x^2\right)}$$

Classically

$$P(x)\,dx \propto \frac{dx}{v} = \frac{dx}{\sqrt{\frac{2}{m}\left(E - \frac{1}{2} m\omega^2 x^2\right)}}$$

Estimate the energy of SHO by uncertainty principle

$$V(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$$

$$(\Delta p)^2 = \overline{p^2} - \underbrace{(\bar{p})^2}_{\substack{\| \\ 0}} = \overline{p^2}$$

(why?)

$$(\Delta x)^2 = \overline{x^2} - \underbrace{(\bar{x})^2}_{\substack{\| \\ 0}} = \overline{x^2}$$

(why?)

$$\therefore \quad \Delta p \, \Delta x \geq \hbar \quad \Rightarrow \quad \overline{p^2} \geq \frac{\hbar}{\overline{x^2}}$$

Estimate $\quad E = \dfrac{\hbar^2}{2mx^2} + \dfrac{1}{2}m\omega^2 x^2$

$$0 = \frac{dE}{dx} = -\frac{\hbar^2}{mx^3} + m\omega^2 x$$

$$\Rightarrow \quad x^2 = \frac{\hbar}{m\omega}$$

$$\therefore \quad E = \frac{\hbar^2}{2m\left(\frac{\hbar}{m\omega}\right)} + \frac{1}{2}m\omega^2 \frac{\hbar}{m\omega}$$

$$= \frac{\hbar\omega}{2} + \frac{\hbar\omega}{2} = \hbar\omega$$

off by a factor of 2

In general, QM in 1D:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\psi(x)$$

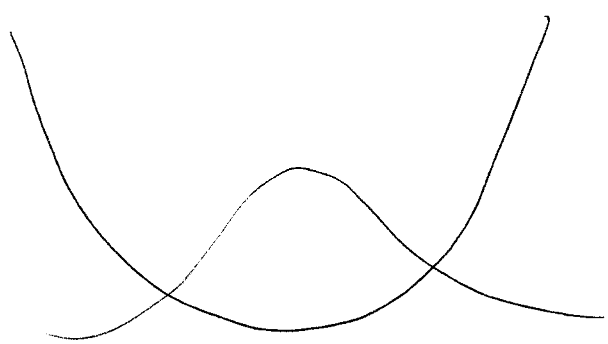
If $V(x)$ is symmetric, i.e.

$V(x) = V(-x)$ [SHO is an example]

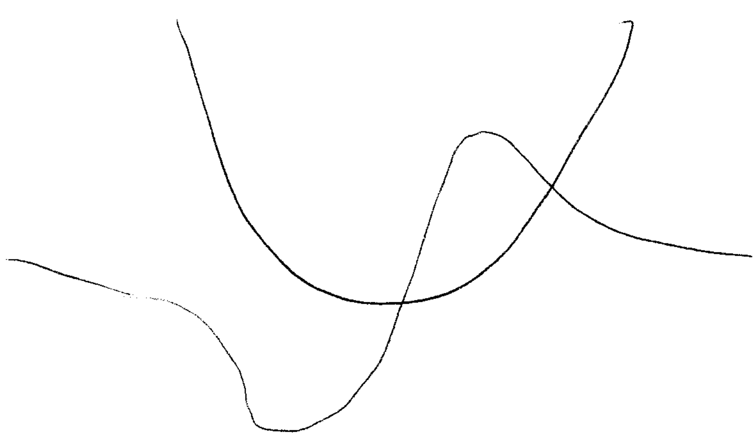
$\Rightarrow$ $|\psi(x)|^2$ must be symmetric

$\Rightarrow$ $\psi(x) = \pm\psi(-x)$ even, odd parity

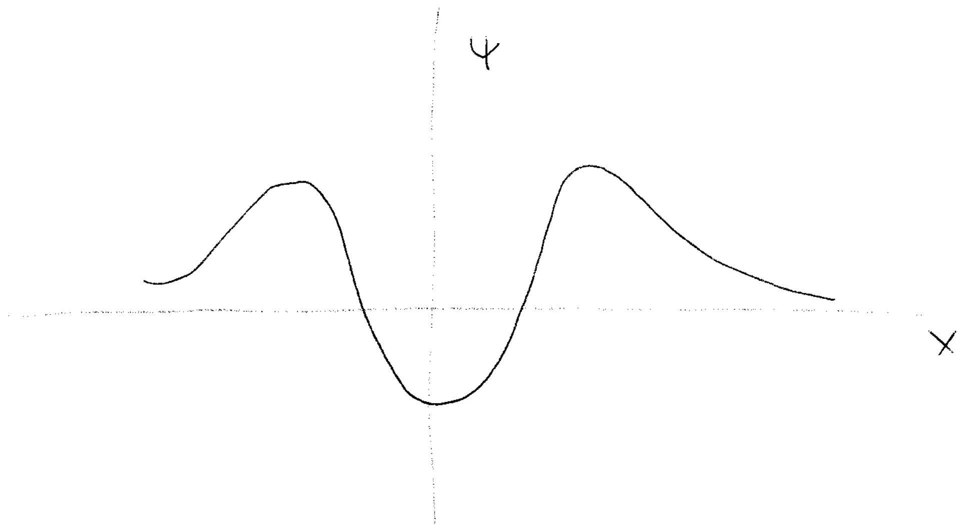
$n=0$



$n=1$



$n=2$

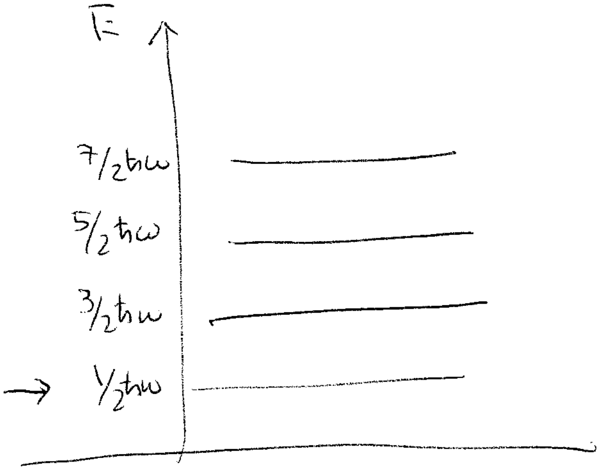


Note that $\psi(0)=0$ for $n=1,3,5,\cdots$

$\psi(0)=$ peak $n=0,2,4,\cdots$

$$\boxed{\begin{aligned} &\text{Energy level} \quad E_n=\left(n+\tfrac{1}{2}\right)\hbar\omega \\ &\psi_n = C_n e^{-\frac{m\omega x^2}{2\hbar}} \underbrace{H_n(x)}_{\text{Hermite polynomial of degree } n} \end{aligned}}$$

will solve this in Phy 448



zero-point energy