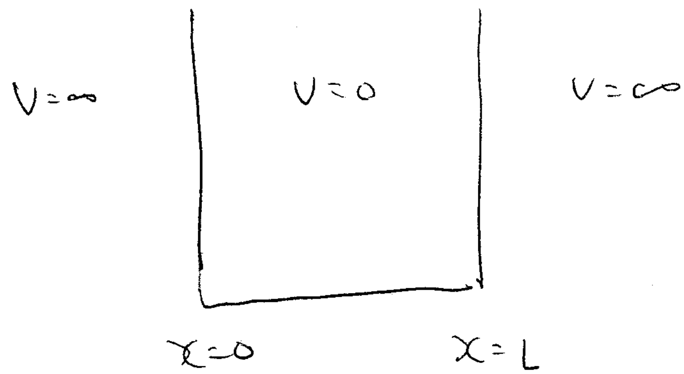
Physics 248 Lecture 15 Gary Shiu 3/2/07

Particle in a box



Try $\Psi(x,t) = f(x)\, e^{-iEt/\hbar}$ (recall standing wave)

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

$$e^{-iEt/\hbar}\left[-\frac{\hbar^2}{2m}\frac{d^2 f}{dx^2} + V f(x)\right] = i\hbar\left(-\frac{iE}{\hbar}\right) e^{-iEt/\hbar} f(x)$$

$$\Rightarrow -\frac{\hbar^2}{2m}\frac{d^2 f(x)}{dx^2} + V f(x) = E f(x)$$

Time-independent Schrodinger equation

[Good for any $V(x)$, not $V(x,t)$]

Boundary conditions

where $V = \infty$ $f = 0$

$$\Rightarrow f(0) = f(L) = 0$$

Where $V = 0$

$$-\frac{\hbar^2}{2m}\frac{d^2 f(x)}{dx^2} = E f(x)$$

Solutions are sine and cosine, but cosine does not satisfy the b.c. at $x = 0$

Try $f(x) = \sin kx$

$$0 = f(L) = \sin kL \quad \Rightarrow \quad k = \frac{n\pi}{L} \quad n = 1, 2, \ldots$$

$$\boxed{E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2} \quad n = 1, 2, \ldots$$

Energy is quantized

<u>Uncertainty Principle</u>

$$\Delta p \, \Delta x \geq \frac{\hbar}{2}$$

Let's check this for the infinite potential well

$(\Delta p)^2 = \overline{(p - \bar{p})^2}$ where $\bar{p}$ = average of p

Note: $\bar{\bar{p}}$ = average of $\bar{p}$

$= \bar{p}$ ($\because \bar{p} = \text{const}$)

$$= \overline{p^2 - 2p\bar{p} + \bar{p}^2}$$

$$= \overline{p^2} - 2\bar{p}^2 + \bar{p}^2$$

$$= \overline{p^2} - \bar{p}^2$$

For particle in a box $\bar{p} = 0$ (why?)

$$\overline{p^2} = \overline{2mE}$$

$$= 2m \frac{\pi^2 n^2 \hbar^2}{2mL^2} \qquad n = 1, 2 \ldots$$

$$= \frac{n^2 \pi^2 \hbar^2}{L^2}$$ for ground state $\{n = 1\}$

$$(\Delta x)^2 = \overline{(x - \bar{x})^2} = \overline{x^2} - \bar{x}^2$$

For partlee in a box $\bar{x} = \frac{L}{2}$ (why?)

$$\bar{x} = \int_{-\infty}^{\infty} x^2 |\psi(x)|^2 \, dx$$

roughly the "width" of $\psi(x)$

$$\overline{x^2} = \int x^2 |\psi(x)|^2 dx$$

$$= \int \psi^*(x)\, x^2\, \psi(x)\, dx$$

First, we need to normalize the wave function

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1 \qquad \text{where } \psi(x) = A \sin \frac{n\pi x}{L}$$

$$\Rightarrow \int_0^L A^2 \sin^2 \frac{n\pi x}{L}\, dx = 1$$

$$\Rightarrow A^2 \int_0^L \left(\frac{1}{2} - \frac{1}{2}\cos \frac{2n\pi x}{L}\right) dx = 1$$

$$\Rightarrow \frac{A^2 L}{2} = 1 \quad \Rightarrow \quad A = \sqrt{\frac{2}{L}}$$

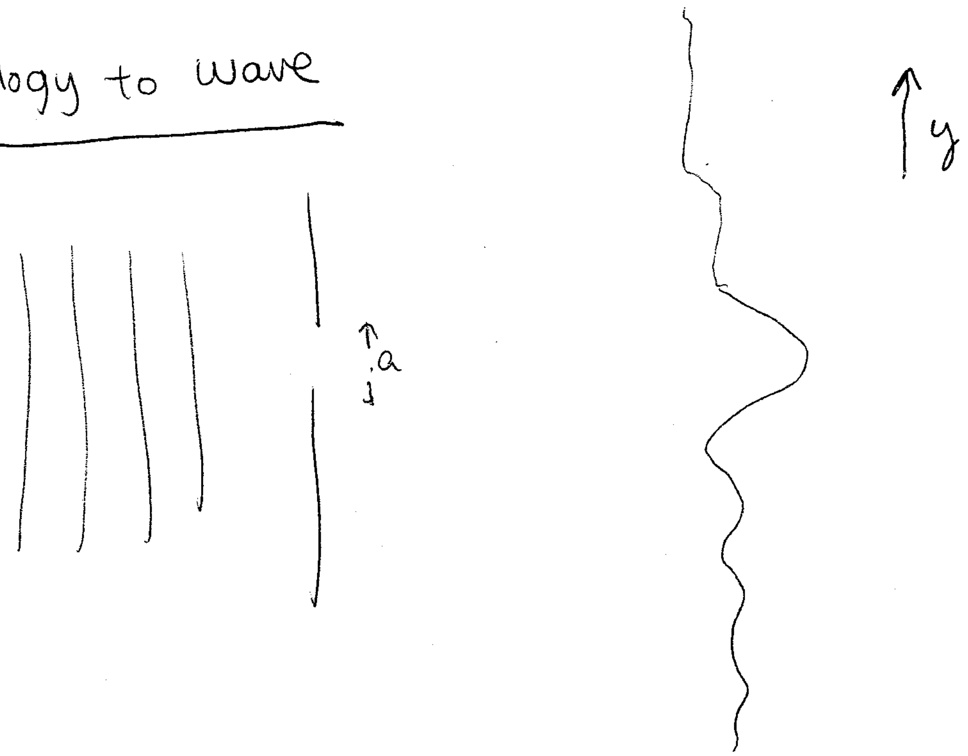
$$(\Delta x)^2 = \overline{(x - \bar{x})^2} = \overline{\left(x - \frac{L}{2}\right)^2}$$

$$= \int_0^L \frac{2}{L}\left(\sin^2 \frac{n\pi x}{L}\right)\left(x - \frac{L}{2}\right)^2 dx$$

$$= 0.03\, L^2$$

$$\Delta p \Delta x = \frac{\pi\hbar}{L}\sqrt{0.14 L^2} = 0.57\,\hbar \;\geq \frac{\hbar}{2}$$

Analogy to wave



$$k_y = k\Delta\theta = k\frac{\lambda}{a} = \frac{2\pi}{a}$$

$$\hbar k_y a = 2\pi\hbar = h$$

$$p_y a \sim h$$

uncertainty principle

~ wave spread.