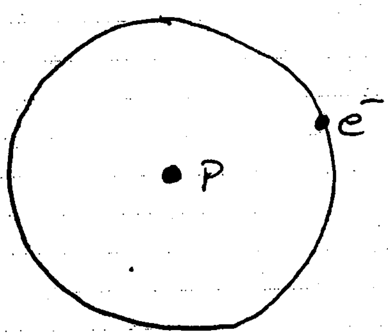
¿ <u>HOW DOES A PARTICLE GO FROM ONE QUANTUM STATE TO ANOTHER?</u>

<u>BOHR:</u> $h\nu_{m} = \underbrace{E_m - E_n}_{\text{ENERGY LEVELS OF QUANTUM SYSTEM.}}$ ← EXPLAINS WHY MATTER IS STABLE.

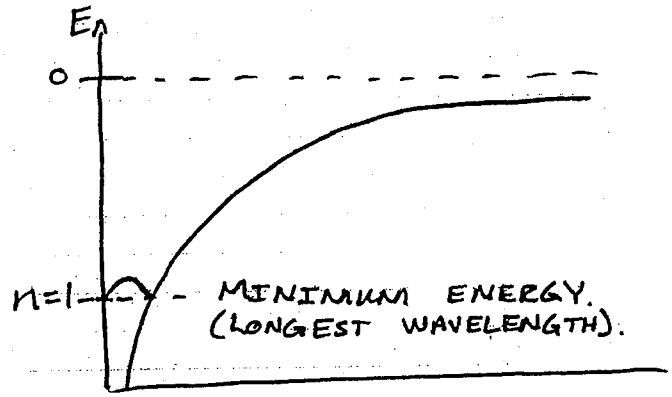
↑ LIGHT FREQUENCY.

(NOT ALWAYS A PHOTON OF LIGHT, SOMETIMES IN SOLID, EMITS PHONON)

↖ VIBRATIONAL WAVE.



$E_{NERGY} = \frac{-ke^2}{r}$



- WILL FIND THAT THERE IS A FINITE PROBABILITY OF THE ELECTRON BEING INSIDE THE NUCLEUS. ~~FOR SOME~~

¿ BY WHAT MECHANISM IS THE PHOTON EMITTED?

→ MODES OF THE ELECTRIC FIELD ARE THE QUANTIZED ELECTROMAGNETIC FIELD (PHOTONS).
→ VACUUM HAS ZERO POINT ENERGY, PHOTONS "BOILING" OUT OF & BACK INTO VACUUM.

¿ FOR $n=2$ PARTICLE IN A BOX, WITH ZERO PROBABILITY OF FINDING THE PARTICLE IN THE MIDDLE, HOW DOES IT GET FROM SIDE TO SIDE?
— ONLY KNOW PROBABILITY. PARTICLE IS ON BOTH SIDES WITH EQUAL PROBABILITY.

- $|\psi|^2$ = PROBABILITY OF FINDING PARTICLE.

↑ (ABSOLUTE SQUARED, OR ABSOLUTE OF SQUARE? WHAT'S DIFF.)

- DERIVATION / DISCOVERY OF THE SCHRÖDINGER EQ$^{\underline{n}}$:
  - SCHRÖDINGER EQ$^{\underline{n}}$ IS NOT UNIQUE, THERE ARE OTHER BEAUTIFUL DESCRIPTIONS, BUT IT IS THE ONE THAT HAPPENS TO PREDICT THE OUTCOME OF EXPERIMENTS.
  - KEY TO SUCCESS: EXACTLY PREDICTED THE ENERGY LEVELS OF THE HYDROGEN ATOM.

- REASONING BEHIND SCHRÖDINGER EQ$^{\underline{n}}$:
  - PLANE WAVE:

$$e^{i\vec{k}\cdot\vec{r}-i\omega t}$$

(PLANE WAVE MOVING IN DIRECTION $\vec{k}$).

KNEW DeBROGLIE RELATIONS:

LIGHT: $\vec{P} = \hbar\vec{k}$ $\quad E = \hbar\omega$.

MATTER: $E = \frac{p^2}{2m} + V(r)$

$$\hbar\omega = \frac{\hbar^2 k^2}{2m} + V$$

$$\Rightarrow i\hbar\frac{\partial\psi}{\partial x} = i\hbar i k\psi$$

$$\Rightarrow \underbrace{-i\hbar\frac{\partial\psi}{\partial x}}_{} = \hbar k\psi = \underbrace{P\psi}_{}.$$

$\Rightarrow$ OPERATOR: $P_{op} = -i\hbar\frac{\partial}{\partial x}$

$$\underbrace{i\hbar\frac{\partial\psi}{\partial t}}_{} = i\hbar(-i\omega)\psi = \underbrace{\hbar\omega}_{\text{ENERGY.}}\psi$$

$$E_{op} = +i\hbar\frac{\partial}{\partial t}.$$

$$\frac{P_{op}^2}{2m}\psi + V\psi = E_{op}\psi$$

$$\Rightarrow \frac{1}{2m}(i\hbar)^2\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\boxed{\therefore\ \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}}$$

- THIS IS A DISCOVERY; A POSSIBLE WAY THAT NATURE MIGHT BEHAVE — THAT HAPPENS TO WORK.
  - ONLY MOTIVATED THE GUESS BY USING

$$\psi = e^{i\vec{k}\cdot\vec{r} - i\omega t}$$

→ BUT ONLY A SOLUTION IF V IS A CONSTANT.

RELATIVITY: $p^2c^2 + m^2c^4 = E^2$

- START TO KLEIN GORDON EQ$^{\underline{n}}$ (RELATIVISTICLY CORRECT) (TURNED OUT TO HAVE PROBLEMS WHEN SCHRÖDINGER WORKED IT OUT).

$$-\hbar^2c^2 \frac{\partial^2\psi}{\partial x^2} + m^2c^4\psi + V\psi = -\hbar^2 \frac{\partial^2\psi}{\partial t^2}$$

BUT THIS ONLY WORKS FOR SOME PARTICLES, NOT ELECTRONS, SO IT'S WRONG FOR ATOMS.

CHECK TO SEE IF $V=0$ GIVES DE BROGLIE RELATIONS:

$$\psi = e^{ikx - i\omega t}$$

$$\Rightarrow \quad \frac{-\hbar^2}{2m}(ik)^2\psi = i\hbar(-\omega)\psi$$

$$\boxed{\frac{\hbar^2k^2}{2m} = \hbar\omega} \quad \checkmark.$$

- BUT WHAT DOES ψ MEAN?:

$|\psi_{(\vec{r})}|^2$ = PROBABILITY OF FINDING PARTICLE AT POINT $\vec{r}$

$$\int_{\text{ALL SPACE}} |\psi_{(\vec{r})}|^2 d^3r = 1 \qquad \text{"NORMALIZATION".}$$

$\psi^*_{(\vec{r})}\psi_{(\vec{r})}$

$$\int \psi^*_{(\vec{r},t)}\psi_{(\vec{r},t)}\, d^3r = 1$$

COPENHAGEN INTERPRITATION.

What replace classical trajectories are "averages" in the probabilistic sense:

expectation values

$$\langle x \rangle = \int_{-\infty}^{\infty} x |\psi|^2 \, dx \qquad \text{average position}$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 |\psi|^2 \, dx \qquad \text{average of } x^2$$

$\neq$ (average of x)^2

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \qquad \text{uncertainty in } x$$

(root mean squared)

Similar for p and E.

Uncertainty principle $\quad \Delta p \, \Delta x \geq \frac{\hbar}{2}$

We will study a few (solvable) systems:

1. Particle in a box
2. Simple Harmonic Oscillator
3. Hydrogen atom