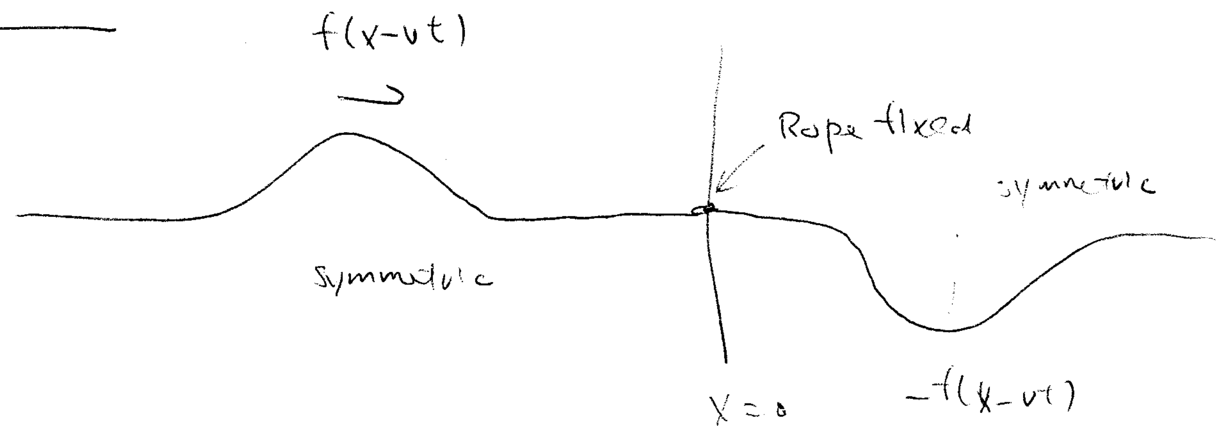
Superposition Principle:

If f_1 & f_2 are waves

$\Rightarrow$ $Af_1 + Bf_2$ is also a wave (check solutions!)

Reflection



Boundary condition

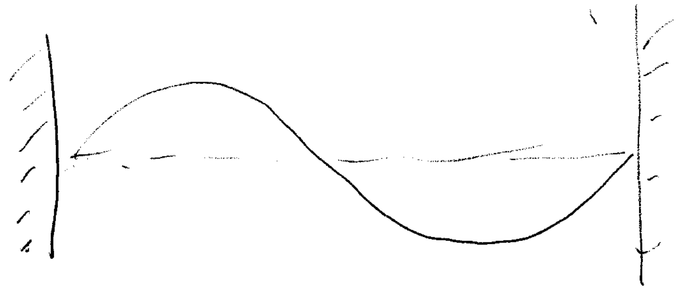
$$f(x=0, t) = 0$$

$$f_{tot} = f(x-vt) - f(x+vt)$$

$$f_{tot}(0,t) = f(-vt) - f(-vt) = 0$$

Idea of "images": Replace boundary w/
second wave that mimics boundary

Standing waves (or bounded waves)



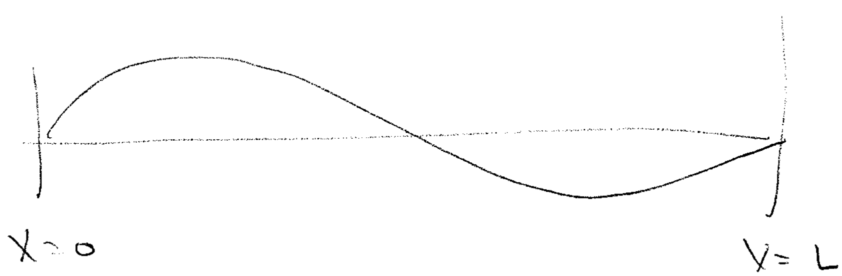
$$\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}$$

Assume $f = g(x)\, e^{-i\omega t}$

(Not restrictive because of superposition)

$$\sim (-i\omega)^2 g e^{-i\omega t} = v^2 \frac{\partial^2 g}{\partial x^2} e^{-i\omega t}$$

$$\frac{\partial^2 g}{\partial x^2} = -\frac{\omega^2}{v^2} g = -k^2 g$$



$$g = A \sin kx + B \cos kx$$

$$g(0) = 0 \Rightarrow B = 0$$

$$\therefore\ g(x) = A \sin kx$$

$$g(L) = 0 \Rightarrow kL = n\pi \qquad n \in \mathbb{Z}$$

$$\Rightarrow \lambda = \frac{2L}{n} \Rightarrow \frac{\lambda}{2} = \frac{L}{n}$$

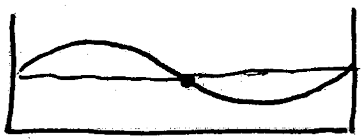
CH. 33:

INTERFERENCE AND DIFFRACTION:

RECALL
STANDING WAVE:

$$\underbrace{e^{ikx-i\omega t}}_{\text{RIGHT GOING WAVE}} + \underbrace{e^{-ikx-i\omega t}}_{\text{LEFT GOING WAVE.}} = e^{-i\omega t}\underbrace{\left(e^{ikx}+e^{-ikx}\right)}_{2\cos(kx).}$$

$$= 2e^{-i\omega t}\cos(kx)$$



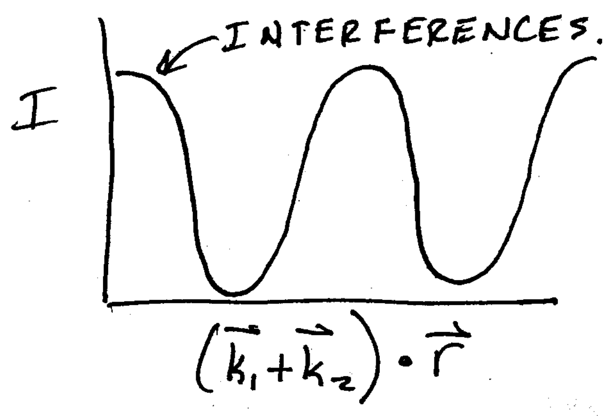
IN GENERAL: HAVE 2 WAVES TRAVELLING IN DIFF. DIRECTIONS:

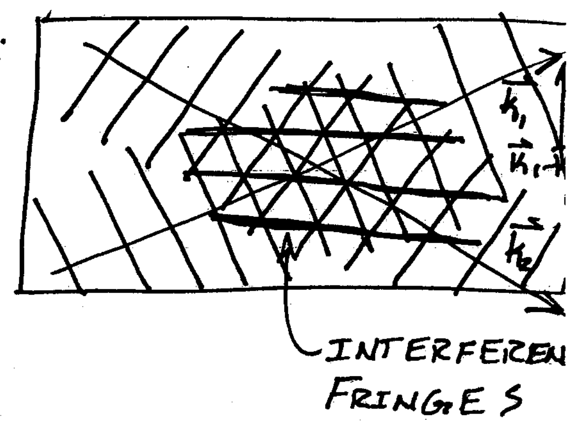
$$e^{i\vec{k}_1\cdot\vec{r}-i\omega t} + e^{i\vec{k}_2\cdot\vec{r}-i\omega t} = e^{-i\omega t}\left(e^{i\vec{k}_1\cdot\vec{r}}+e^{i\vec{k}_2\cdot\vec{r}}\right)$$

$$= e^{-i\omega t - \frac{\vec{k}_1+\vec{k}_2}{2}\cdot\vec{r}\, i}\left(e^{i\left(\frac{\vec{k}_1+\vec{k}_2}{2}\cdot\vec{r}\right)} + e^{i\left(\frac{\vec{k}_1+\vec{k}_2}{2}\right)\cdot\vec{r}}\right)$$

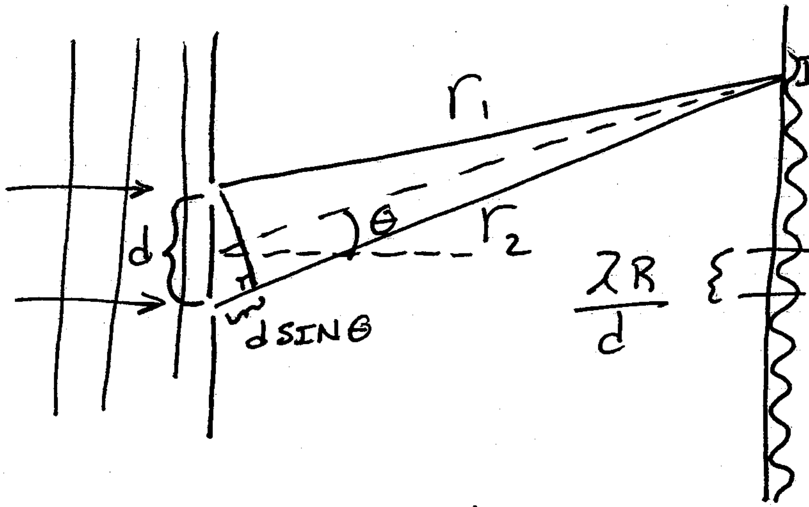
$$= e^{-i\omega t}\cdot e^{i\frac{\vec{k}_1+\vec{k}_2}{2}\cdot\vec{r}}\, 2\cos\left(\frac{\vec{k}_1-\vec{k}_2}{2}\cdot\vec{r}\right)$$

INTENSITY: $|f|^2 = 4\cos^2\left(\frac{\vec{k}_1-\vec{k}_2}{2}\cdot\vec{r}\right)$





Double Slit Diffraction:



$$f = e^{ik r_1} + e^{ik r_2}$$

$$I = 2 + 2\cos[k(r_1 - r_2)]$$

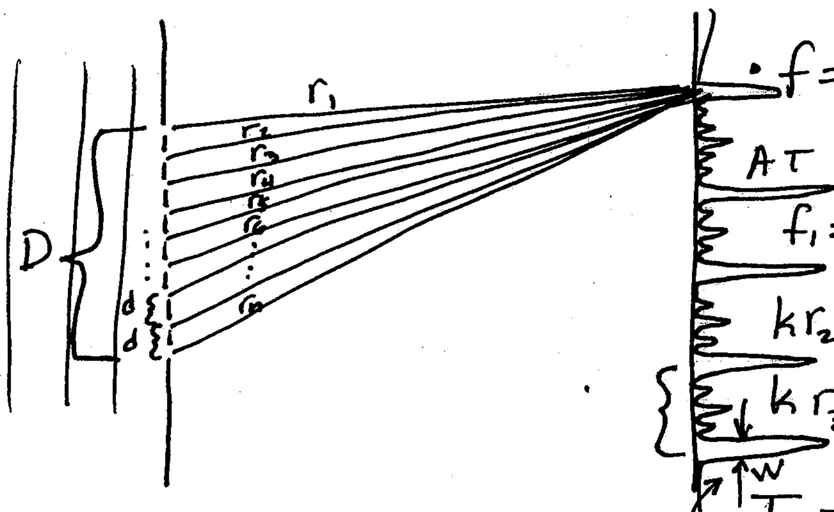
$$k(r_1 - r_2) = n(2\pi)$$

"Constructive interference"

$$\frac{2\pi}{\lambda}(r_1 - r_2) = 2\pi n \qquad \underline{r_1 - r_2 = n\lambda}.$$

$$\Rightarrow \quad d\sin\theta = n\lambda.$$

Many slits: n - slits.



$$f = f_1 + f_2 + f_3 + \cdots + f_n$$

At maximum

$$f_1 = f_2 = f_3 = \cdots = f_n.$$

$$kr_2 = kr_1 + 2\pi$$

$$kr_3 = kr_2 + 2\pi = kr_1 + 4\pi.$$

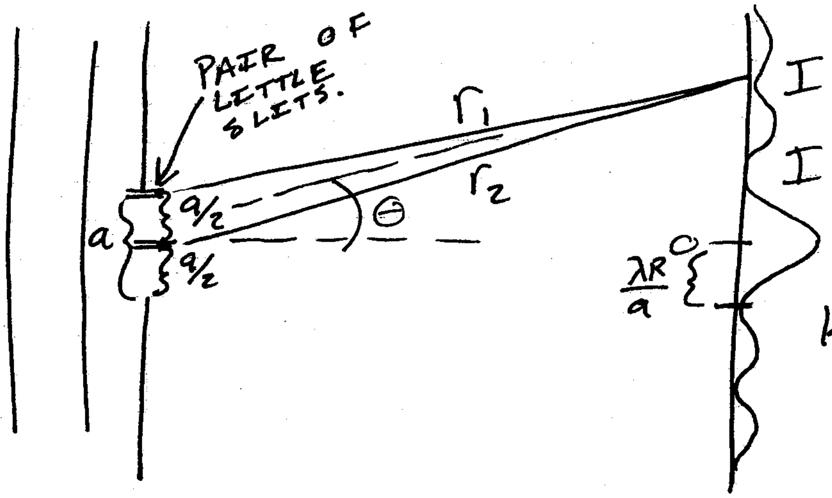
$$I = |f|^2 = N^2 f_1^2$$

Number of slits. Why not n?

What width w to conserve energy.

$$\cancel{n} I_1 = n^{\cancel{2}} \cancel{f_1^2} \cdot \frac{w}{\frac{\lambda R}{d}} \qquad W = \frac{\lambda R}{nd} = \frac{\lambda R}{D}$$

SINGLE SLIT:



$I = 0$ IF

$$r_1 + \frac{\lambda}{2} = r_2$$

$$kr_2 = kr_1 + \pi.$$

$$\frac{2\pi}{\lambda} r_2 = \frac{2\pi}{\lambda} r_1 + \pi$$

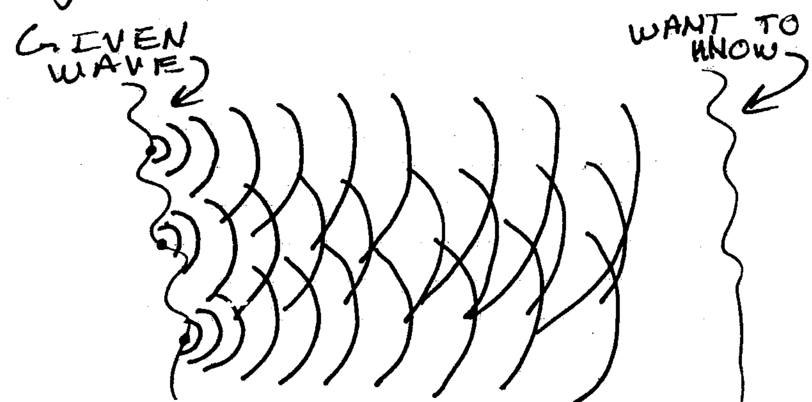
$$r_2 - r_1 = \frac{\lambda}{2}$$

$$\Rightarrow \frac{a}{2} \sin\theta = \frac{\lambda}{2}$$

WHERE IS $I = 0$?

$I = 0$ WHEN $\sin\theta = \frac{\lambda}{a}$.

Huygen's PRINCIPLE:



THING OF EACH pt. ON A WAVE AS A SOURCE OF SPHERICAL WAVES.

- WORKS TO A VERY GOOD APPROXIMATION.

$$I = \left(\frac{\sin \frac{ka}{2} \sin\theta}{\frac{ka}{2} \sin\theta} \right)^2$$