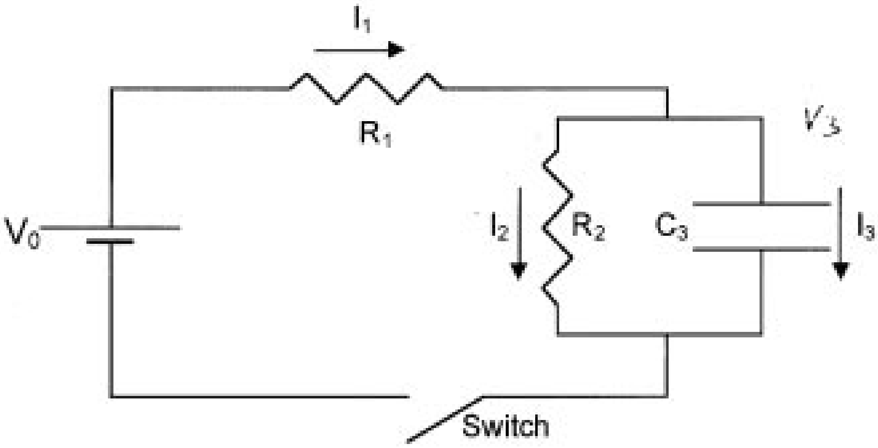
Name Solutions

Exam #3
Physics 248
April 28, 2004

Each problem is worth 25 points

Problem	Score
1	
2	
3	
4	
Total	

1. The circuit below is described by the three currents shown, each of which is a function of time. At t=0, the charge on the capacitor is zero, and the switch is suddenly closed.



(a) What are the three currents I_1, I_2, and I_3 at $t = 0$, right after the switch is closed?

7

$V_3 = 0 \Rightarrow I_2 = 0$

$I_1 = I_3 = V_0/R_1$

(b) What are the asymptotic values of I_1, I_2, and I_3 as $t \to \infty$?

6

$I_3 = 0 \quad I_1 = I_2 = \frac{V_0}{R_1 + R_2}$

(c) Using Kirchoffs loop law for voltages twice, and Kirchoffs current law once, write three equations that could be solved for I_1, I_2, and I_3 as a function of time. Note that one or two of these equations will be differential, but not all will!

10

$V_0 - I_1 R_1 - I_2 R_2 = 0$

$V_0 - I_1 R_1 - q_3/C_3 = 0$ 8

$I_1 = I_2 + I_3$

$q_3 = \int_0^t I_3 \, dt$

2. Assume the potential energy between an electron and a proton is given by $V(r) = m\omega_0^2 r^2/2$ instead of the usual Coulombs law. Using the quantization of angular momentum in units of $\hbar$, derive the energies of the allowed energy levels, as Bohr would have done.

7 $\quad E = \frac{1}{2} m v^2 + \frac{1}{2} m \omega^2 r^2$

7 $\quad \frac{m v^2}{r} = m \omega_0^2 r$

7 $\quad m v r = n \hbar$

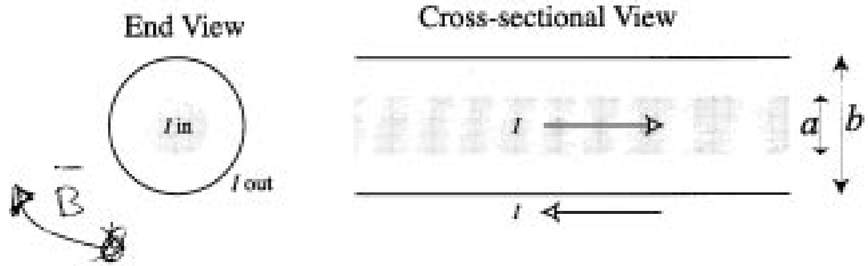
$$m^2 v^2 r^2 = m^2 (\omega_0^2 r^2) r^2 = n^2 \hbar^2$$

$$r^2 = \frac{n\hbar}{m\omega_0}$$

4 $$v^2 = \omega_0^2 r^2 = \frac{n\hbar\omega_0}{m}$$

$$E = \frac{1}{2} m \frac{n\hbar\omega_0}{m} + \frac{1}{2} m\omega_0^2 \frac{n\hbar}{m\omega_0} = n\hbar\omega_0$$

3. A cross-section of a coaxial cable is shown. It consists of an inner cylindrical wire of radius a and an outer thin conducting "sheath" of radius b. A current I, uniformly spread throughout the wire, flows through the inner wire, and returns through the sheath. Calculate the magnetic field for a)$r < a$, b)$a < r < b$, c)$r > b$



a) $r<a$ $\quad B 2\pi r = \mu_0 I \frac{r^2}{a^2}$

$$\boxed{B = \frac{\mu_0 I r}{2\pi a^2}}$$

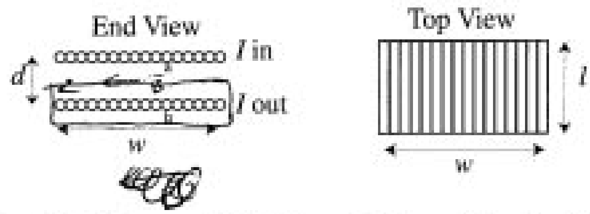
b) $a<r<b$ $\quad B 2\pi r = \mu_0 I$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

c) $r>b$ $\quad B 2\pi r = 0$

$$\boxed{B = 0}$$

4. Shown below is a "parallel-plate" inductor, consisting of two sets of N wires, each wire carrying time-dependent current $I(t)$. For the top plate, the currents flow into the page, while for the bottom plate the currents flow out of the page. The width of the plates is w, the length l, and the distance between them $d \ll w, l$.



(a) Use Ampere's law to find the magnetic field (magnitude and direction) in between the two plates. You may assume the field is zero everywhere except between the plates.

$$B \cdot w = \mu_0 N I$$

$$B = \mu_0 \frac{NI}{w}$$ pointing to left

(b) Calculate the induced emf for a single pair ab of the wires, and for the whole system.

$$\mathcal{E}_{ab} = -\frac{d\Phi_m}{dt} = -\frac{\mu_0 N \dot{I}}{w} \cdot l d$$

$$= -\frac{\mu_0 N \dot{I} l d}{w}$$

$$\mathcal{E} = -\frac{\mu_0 N^2 \dot{I} l d}{w}$$

(c) Calculate the inductance.

$$\mathcal{E} = -L\frac{dI}{dt} \Rightarrow L = \frac{\mu_0 N^2 l d}{w}$$