Name Solutions

Exam #3

Physics 248

April 26, 2006

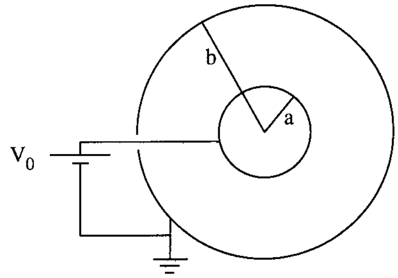
Each problem is worth 25 points

Problem	Score
1	
2	
3	
4	
Total	

Useful math: $\sin 2\theta = 2\cos\theta\sin\theta$ | $\cos 2\theta = \cos^2\theta - \sin^2\theta$

1 mi=1.609 km | $\sin\theta \approx \theta$ | $\cos\theta \approx 1 - \theta^2/2$

1. A pair of long concentric conducting cylinders of length l and radii a and b are attached to a battery of voltage V_0. The outer cylinder is grounded.



(a) Use Gauss's law to find the potential everywhere. Express your answer in terms of V_0.

$r<a \quad V=V_0 \qquad r>b \quad V=0$

$a<r<b \qquad E\cdot 2\pi r l = \frac{Q}{\epsilon_0} \Rightarrow E = \frac{Q}{2\pi r l \epsilon_0} \Rightarrow V(r)-V(a) = -\int_a^r E\cdot dl$

$= \frac{-Q}{2\pi l \epsilon_0} \ln r/a$

$V(r) = V_0 - \frac{Q}{2\pi l \epsilon_0} \ln r/a \qquad V(b)=0 \Rightarrow \frac{Q}{2\pi\epsilon_0 l} = \frac{V_0}{\ln b/a}$

$\therefore V(r) = V_0\left(1 - \frac{\ln r/a}{\ln b/a}\right)$

(b) Find the capacitance.

$C = \frac{Q}{V_0} = \frac{2\pi\epsilon_0 l}{\ln b/a}$

(c) Find the electrostatic energy.

$U = \frac{1}{2}CV_0^2 = \frac{1}{2}\left(\frac{2\pi\epsilon_0 l}{\ln b/a}\right)V_0^2 = \frac{\pi\epsilon_0 l}{\ln b/a}V_0^2$

(d) A dielectric material of dielectric constant κ is inserted between the cylinders. Find the energy.

V_0 is fixed $\Rightarrow E$ is same.

$\therefore Q \Rightarrow KQ, \quad C \Rightarrow KC$

$\therefore u = \frac{\pi K\epsilon_0 l}{\ln b/a} V_0^2$

2. In some solids, electrons can bind to positively charged "holes", forming so-called exitonic atoms. The effects of the material are that the electrons have an effective mass m^*, the holes also have an effective mass m_h, and the dielectric constant κ of the material must also be accounted for.

a) If $\kappa = 10$, and $m^* = m_h = 1.4m_e$, find the energy levels of the exitonic atom.

$$V(r) = \frac{-ke^2}{Kr} \quad \therefore \quad ke^2 \Rightarrow \frac{ke^2}{K}$$

H $$E_n = -\frac{1}{2}\frac{\alpha^2 m_e c^2}{n^2(1+m/M)} \overset{\text{excaton}}{\Rightarrow} \frac{1}{2}\left(\frac{\alpha}{K}\right)^2 \frac{1.4 m_e c^2}{\left(1+\frac{1.4}{1.4}\right)n^2}$$

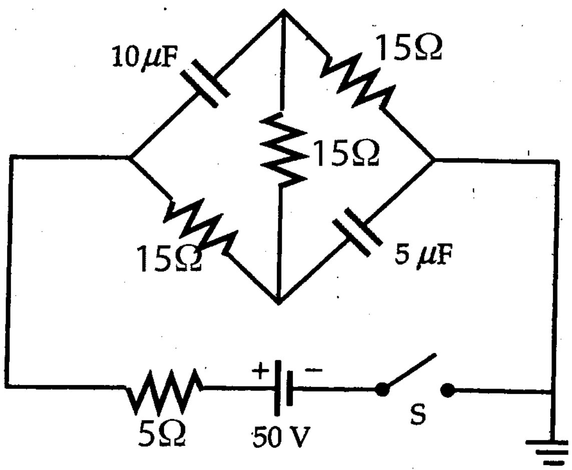
$$= \frac{-13.6\,eV}{10^2 n^2} \cdot 0.7 = \frac{-.095\,eV}{n^2}$$

b) Find the wavelength of a photon emitted by the electron as it decays from $n = 3$ to $n = 2$.

$$\frac{hc}{\lambda} = E_3 - E_2 = .095\,eV\left(\frac{1}{4}-\frac{1}{9}\right) = .013\,eV$$

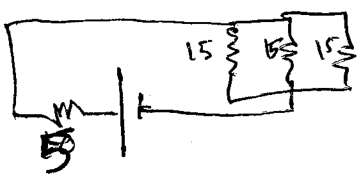
$$\lambda = \frac{2\pi\hbar c}{.013\,eV} = \frac{2\pi(1973\,eV\text{Å})}{.013\,eV} = 9.4\times10^5\,\text{Å}$$

3. The capacitors in the circuit shown are initially uncharged.



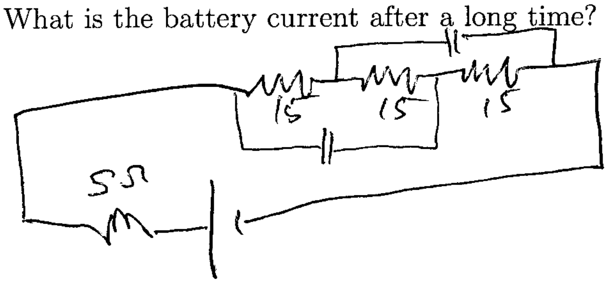
(a) What is the initial value of the battery current when switch S is closed?

Immediately after switch is closed



$$\Rightarrow I = \frac{50\,V}{5\Omega + 5\Omega} = 5\,A$$

(b) What is the battery current after a long time?



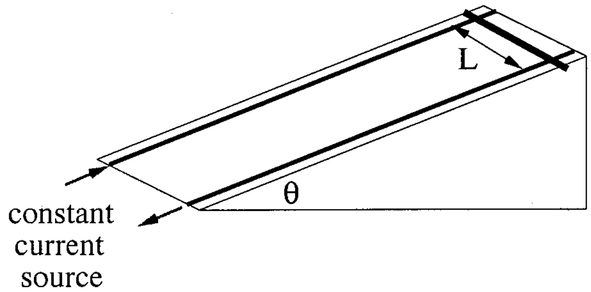
$$I = \frac{V}{50\Omega} = 1A$$

(c) What are the final charges on the capacitors?

$$Q = 10\mu F\,(30\Omega \cdot 1A) = 300\mu C$$

$$Q = 5\mu F\,(30V) = 150\mu C$$

4. A metal crossbar of mass m rides on a pair of long conducting rails mounted on an inclined plane at an angle θ. The rails are separated by a distance L and connected to a device that supplies constant current I to the circuit as shown in the figure. A constant magnetic field is pointing towards the ground.



(a) What is the magnitude of the magnetic field needed to keep the bar from sliding down the rail? Assume that the rails are frictionless.

$\sum F = 0$ along the incline plane $\Rightarrow$ $mg \sin\theta = ILB\cos\theta$

$$B = \frac{mg}{IL} \tan\theta$$

(b) What is the acceleration of the bar if B has twice the value found in part (a)?

$$ILB\cos\theta - mg\sin\theta = ma$$

$$2mg\sin\theta - mg\sin\theta = ma$$

$$a = g\sin\theta$$

up the slope