Name Solutions

Exam #2
Physics 248
March 24, 2004

Each problem is worth 25 points

Problem	Score
1	
2	
3	
4	
Total	

1. For some unknown potential $V(x)$, you are given that the wavefunction for a particular energy eigenstate is $\psi(x) = \psi_0 x \exp(-x^2/2a^2)$. You are relieved to discover in your favorite integral table that $\int_{-\infty}^{\infty} x^{2n} \exp(-x^2)dx = \sqrt{\pi} \cdot 1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)/2^n$.

10   (a) Normalize $\psi(x)$. That is, find ψ_0 such that the integral of $|\psi(x)|^2$ from $-\infty$ to ∞ is 1.

$$\psi_0^2 \int x^2 e^{-x^2/a^2}\, dx \stackrel{5}{=} \psi_0^2 a^3 \underbrace{\int_{-\infty}^{\infty} u^2 e^{-u^2}\, du}_{\sqrt{\pi}/2} = 1$$

$$\psi_0 = \frac{5}{\sqrt{2}}{(\pi)^{1/4} a^{3/2}}$$

15   (b) Calculate $\langle x \rangle$ and $\langle x^2 \rangle$ for the wavefunction $\psi(x)$.

$$\langle x \rangle = 0 \quad \text{by symmetry}$$

$$\langle x^2 \rangle \stackrel{5}{=} \psi_0^2 \int x^4 e^{-x^2/a^2}\, dx$$

$$= \psi_0^2 a^5 \underbrace{\int u^4 e^{-u^2}\, du}_{3\sqrt{\pi}/4}$$

$$= \frac{2}{\sqrt{\pi}\, a^3} a^5 \cdot \frac{3\sqrt{\pi}}{4} = \frac{3a^2}{2}$$

2. The figure below shows the potential energy $V(x)$ for a particle.

   (a) (20 pts) Assuming that the two lowest energy eigenstates have energies as labeled, plot the wavefunctions for these two states, using the dotted lines as the zero levels for the wavefunctions. Mark the classical turning points.

Nodes   5,5
C. T. P.   5
Curvature   5

E_1

$\langle x \rangle$

E_0

x=0
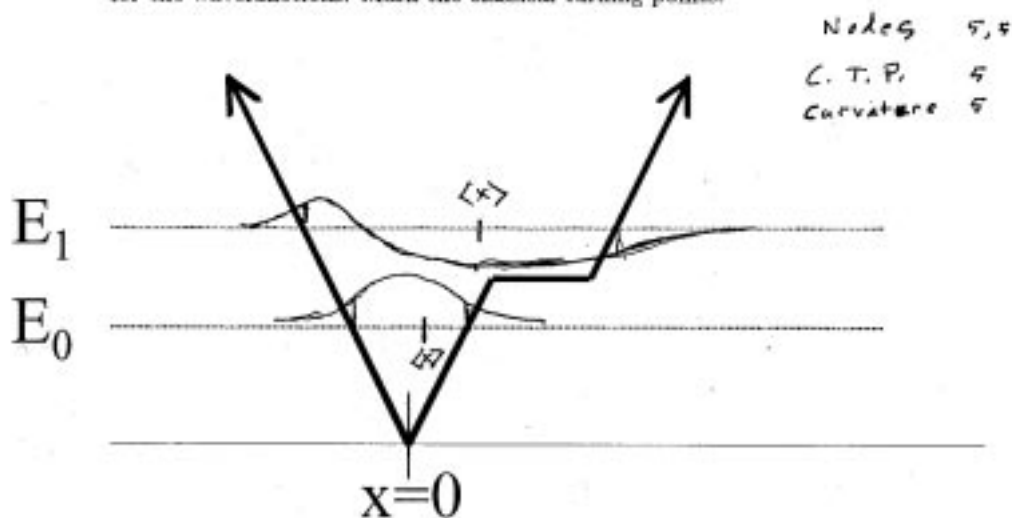
   (b) (5 pts) On the graph, mark your best guess for the position of $\langle x \rangle$ for both of these wavefunctions, and explain the logic behind your decisions.

The tunneling will penetrate further on the right side, giving a slight preference for right over left.

For the 1st excited state, the low kinetic energy on the right further favors the particle being there.

3. Two very long lines of length L and charge $\pm Q$ are a distance d apart, as shown.

(a) Use Gauss's Law to find the electric field due to the positively charged line at the position of the negatively charged line.

$$E \, 2\pi d L = 4\pi k Q$$

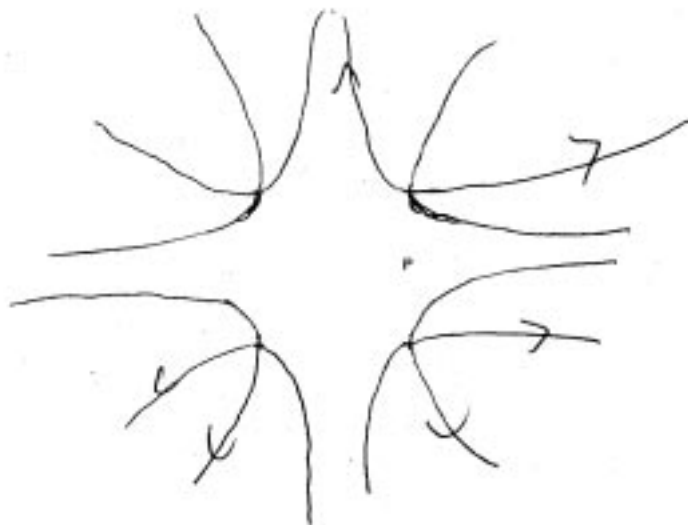$$E = \frac{2kQ}{Ld} \qquad \text{pointing "down"}$$

(b) Calculate the force on the negatively charged line.

$$F = \int E \, dq = - \frac{2kQ}{Ld} \cdot Q$$

$$= - \frac{2kQ^2}{Ld}$$

$$\text{points "up"}$$

each of charge +q

4. A set of 4 positive charges is shown, located at the corners of a cube of side a.

   (a) Draw the electric field lines for this configuration. Use 4 lines per charge.

   (b) A dipole of moment p sits at position P that lies halfway between the two charges on the right. It is oriented vertically on the page. Find the magnitude of the torque on the dipole. What direction will it rotate?

$$E \text{ at } P: \qquad E_x = \frac{2kq}{\sqrt{(5a^2/4)}} \cdot \frac{a}{\sqrt{5a^2/4}}$$

$$\sqrt{(\tfrac{q}{2})^2 + a^2}$$

$$= \frac{2kq}{(5/4)^{3/2} \, a^2}$$

$$|\tau| = p \, E \sin\theta \atop {}_{\uparrow 1} \; = \; \frac{2kqp}{(5/4)^{3/2} a^2}$$

rotates ~~counter~~ clockwise