Name Solutions

# Exam #2

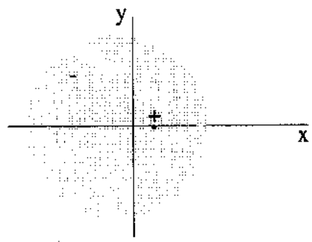
## Physics 248

## March 28, 2001

Each problem is worth 25 points

| Problem | Score |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| Total | |

1. Shown below is a positive point charge $e$ at $x = a/2$ and a uniform negative spherical charge density of total charge $-e$ centered at $x = -a/2$. This charge distribution produces an electric field outside the sphere that looks like a dipole field.




(a) Make a qualitative sketch of the electric field lines outside the sphere.


(b) Use a Taylor series to calculate the x-component of the electric field at the position $y = 0$, $x \gg a$ (outside the sphere).

$$E_x = \frac{ke}{(x-a/2)^2} - \frac{ke}{(x+\frac{a}{2})^2} = \frac{ke}{x^2}\left(\left(1+\frac{2a}{x}\right) - \left(1-\frac{2a}{x}\right)\right)$$

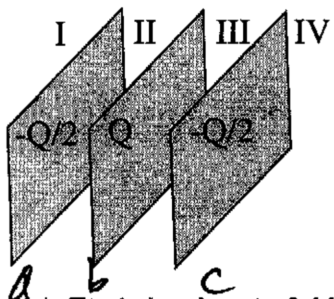
$$= \frac{2kea}{x^3}$$

(c) In class we found that a point dipole has $E_x = 2kp/x^3$. Compare this formula to your field you just calculated to find $p$. Is it what you expected and why?

$p = ea$

Sphere looks like pt. charge

2. Three large conducting plates of area $A$, separated by distance $d$, are aligned as shown. A charge $Q$ is placed on the center plate, with the outside plates having charge $-Q/2$ each.




(a) Find the electric field in each of the four regions of space.

$$E_{I} = E_{IV} = 0$$

$$E_{II} = -4\pi k\left(\frac{Q}{2A}\right) = -2\pi k\frac{Q}{A}$$

$$E_{III} = 2\pi k \, Q/A$$

(b) Find the potential difference between each pair of plates.

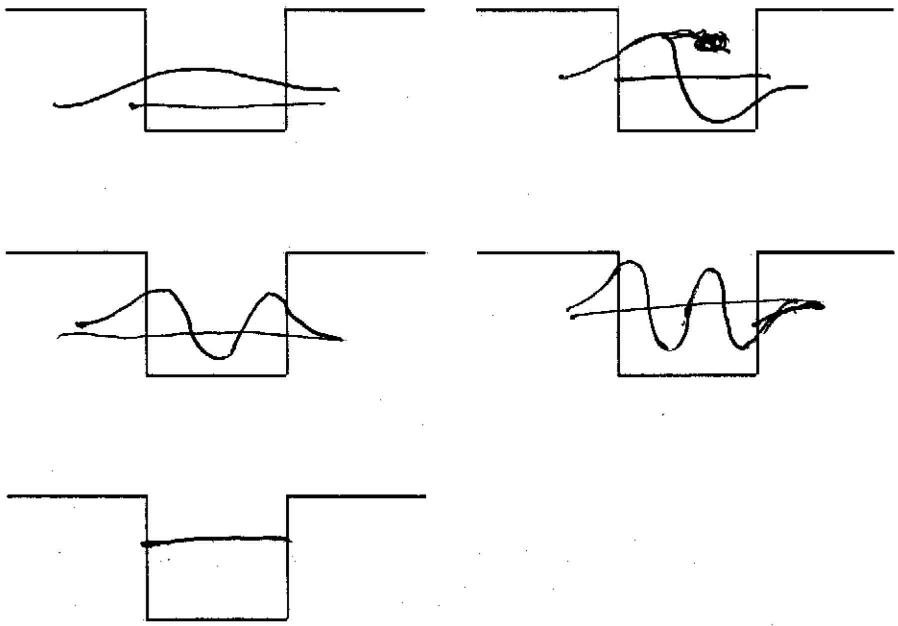
$$V_b - V_c = \frac{2\pi Q d}{A} = V_b - V_c$$

$$V_c - V_a = 0$$

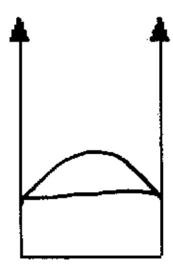
(c) The outer two plates are connected by a wire. Find the capacitance.

$$C = \frac{Q}{V} = \frac{QA}{2\pi Q d k} \Rightarrow C = \frac{A}{2\pi k d}$$

3. (a) Sketch the wavefunction of the four lowest energy states in the finite square well shown below. Show the zero line for the wavefunction in each case. Dont forget to get the curvature correct. In the fifth figure, draw the classical probability distribution for a particle trapped in the finite square well.



(b) Sketch the wavefunction for the ground state of the infinite square well. Is $\langle p^2 \rangle$ bigger or smaller for the ground state in the infinite square well compared with the finite square well? Why?



$$\langle p^2 \rangle = 2mE = \frac{\pi^2 \hbar^2}{L^2}$$

$\langle p^2 \rangle$ is bigger because $x^2$ is smaller

$$\Delta p = \sqrt{\langle p^2 \rangle} \approx \frac{\hbar}{\sqrt{\langle x^2 \rangle}}$$

4. The wavefunction for a particle of mass m is $\psi(x) = \psi_0 e^{-\frac{|x|}{2a}}$, for some constant $a$, and where $-\infty < x < +\infty$ . FYI: $\int_0^\infty x^n e^{-x} dx = n!$ , for any integer n.

(a) Normalize this wavefunction. That is, find the coefficient $\psi_0$ such that the probability of finding the particle between $\pm\infty$ is one.

$$\psi_0^2 \, 2 \int_0^\infty e^{-x/a} dx = \psi_0^2 \cdot 2a = 1$$

$$\psi_0 = \frac{1}{\sqrt{2a}}$$

(b) Assuming $E = \frac{-\hbar^2}{8ma^2}$ , use the time independent Schrodinger equation to show that $V(x)$ is constant for $x > 0$ and $x < 0$ , and find this constant. Don't worry about $V(x)$ at $x = 0$ .

$$-\frac{\hbar^2}{2m}\psi'' - E\psi = V\psi = \left[\frac{-\hbar^2}{2m}\left(\frac{1}{4a^2}\right) - \left(\frac{-\hbar^2}{8ma^2}\right)\right]\psi = 0\psi$$

$$\therefore V = 0$$

(c) Calculate $\langle x^2 \rangle$ and $\langle x \rangle^2$ .

$\langle x \rangle = 0$ by symmetry

$$\langle x^2 \rangle = \frac{a^3}{2a} 2 \int_0^\infty e^{-x/a} \frac{x^2 dx}{a^3} = 2a^2$$

Comment: The actual potential V(x) for this wavefunction is $V(x) = \frac{-\hbar^2}{2ma}\delta(x)$ . That is, the potential is a delta function at the origin that traps the particle, and the potential is zero everywhere else.