Name Solutions

Exam #2

Physics 248

March 29, 2006

Each problem is worth 25 points

Problem	Score
1	
2	
3	
4	
Total	

curve $= 75 + (\text{raw} - 45)\frac{15}{2?}$

Useful math: $\boxed{\sin 2\theta = 2\cos\theta\sin\theta}$ $\boxed{\cos 2\theta = \cos^2\theta - \sin^2\theta}$

$\boxed{1\ \text{mi}=1.609\ \text{km}}$ $\boxed{\sin\theta \approx \theta}$ $\boxed{\cos\theta \approx 1 - \theta^2/2}$

1. A beam of ^{87}Rb atoms (rest energy 81 GeV), moving at 4.59 m/s, passes through a narrow pair of slits (A,B) that are 0.1 mm apart. A distance of 1 m away sits a detector with a narrow slit in front of it. The detector measures the atom current transmitted through the detector slit. The detector slit is moveable along the y-direction.

(a) Calculate the wavelength of the atoms.

$$\lambda = \frac{h}{p} = \frac{2\pi\hbar c c}{m v c^2} = \frac{2\pi(1973\,eV\,\text{Å})(3\times10^8\,m/s)}{81\times10^9\,eV\,(4.59\,m/s)} = 10\,\text{Å}$$

(b) The detector slits is positioned for maximum transmission of atoms, and a current I_0 of atoms is measured. Slit A is then blocked. What current is measured?

$$I \propto (E_A + E_B)^2$$

$$\therefore\ I = \frac{1}{4} I_0$$

(c) With A still blocked, about how far would the detector slits need to be moved for the current to decrease another factor of 2? Slit B is 0.01 mm wide.

first zero at $y = \frac{\lambda R}{b}$

$\therefore$ need to move $\frac{\lambda R}{2b} = \frac{10\text{Å}(1m)}{2(10^{-5}m)} = 5\times10^5\,\text{Å} = 5\times10^{-5}\,m$

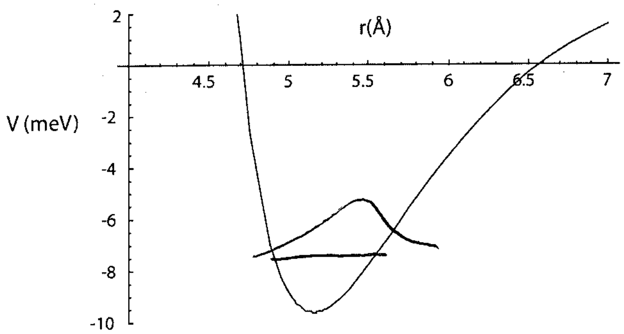
(d) A is now unblocked. In order to get more signal, you decide to replace the detector slit with a pair of slits spaced by a distance s, so that the two slits line up with consecutive interference fringes. What is the value of s needed to observe maximum interference as the slits are moved along y?

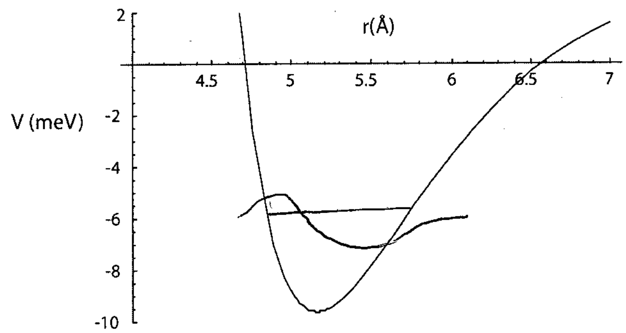
$$n\lambda = d\theta$$

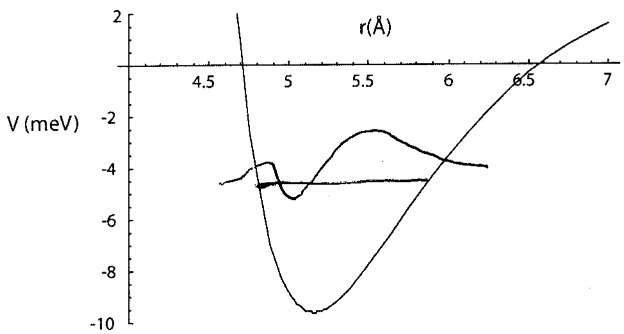
$$\Delta n\,\lambda = d\,\Delta\theta = d\frac{\Delta s}{R} \Rightarrow \Delta s = \frac{R\lambda}{d} = \frac{10\text{Å}(1m)}{10^{-4}m} = 10^{-5}\,m$$

1

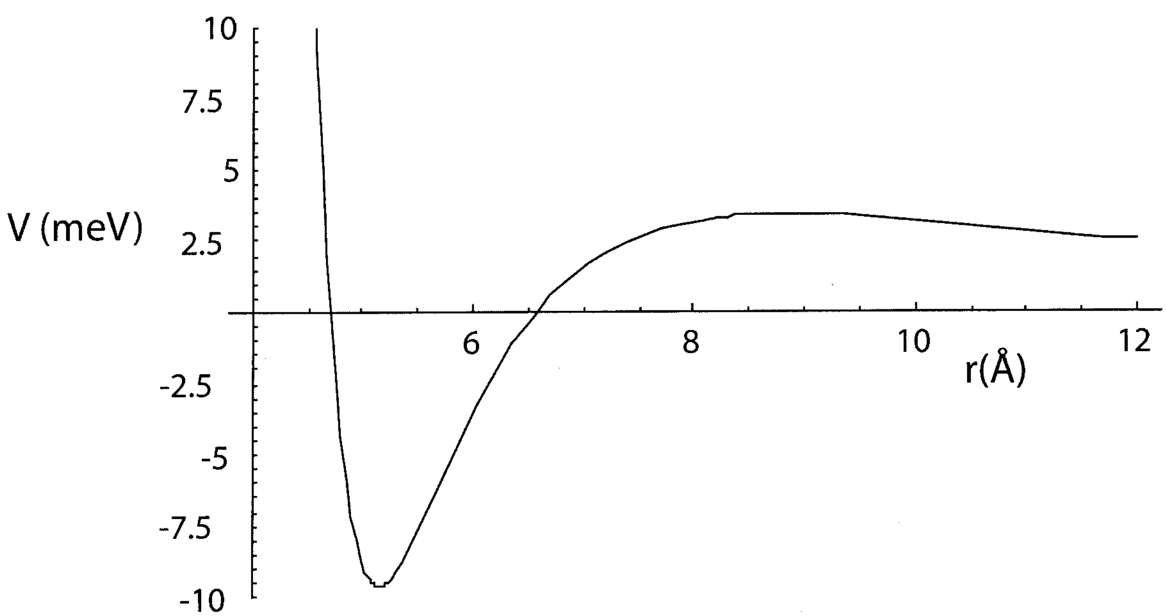
2. (a) (15 pts) A portion of the potential energy curve for a (rotating) Xe_2 molecule is shown below. Sketch your best guess for the wavefunctions of the ground state and the first two excited states.







(b) (10 pts) An expanded version of the same curve is shown here. For $r > 12$ Å the potential continues to decrease towards zero. In what energy region would tunneling cause the molecule to fall apart? At what energies would the tunneling be greatest, and least?



tunnelling $0 < E < 3$ meV

least at 0, most at 3

3. A particle of mass m is confined in the potential $V(x < 0) = \infty$, $V(x > 0) = \alpha x$, where α is a constant. Use uncertainty principle arguments to estimate the zero point energy.

$$\frac{p^2}{2m} + \alpha \underbrace{x}_{h/p} = E$$

$$\frac{p}{m} - \frac{\alpha h}{p^2} = 0 \Rightarrow p^3 = m\alpha h$$

$$\frac{(m\alpha h)^{2/3}}{2m} + \alpha \frac{h}{(m\alpha h)^{1/3}} = E$$

$$\frac{(\alpha h)^{2/3}}{m^{1/2}} \left(1 + 1/2\right) = E$$

$$3/2 \, \frac{(\alpha h)^{2/3}}{m^{1/3}} = E$$

4. Consider an infinitely long solid cylinder of radius R, with a charge per unit length λ.

(a) Find the electric field everywhere if the cylinder is conducting.

$r < R \quad E = 0$

$E \cdot 2\pi r = \lambda \, 4\pi k$

$E = \dfrac{2\lambda k}{r}$

(b) Is the electric field continuous across the surface of the cylinder for (a)? Be quantitative.

discontinuous $\quad \Delta E = \dfrac{2\lambda k}{R}$

(Check $\quad 4\pi k\sigma = 4\pi k\left(\dfrac{\lambda}{2\pi R}\right) = \dfrac{2\lambda k}{R} \quad \checkmark$)

(c) Find the electric field everywhere if the cylinder is a nonconductor and the volume charge density is is nonuniform, given by $\rho(r) = \frac{3\lambda r}{2\pi R^3}$. How does your answer to (b) change?

$r < R$

$E \cdot 2\pi r = 4\pi k \displaystyle\int_0^r 2\pi r\,dr \left(\frac{3\lambda r}{2\pi R^3}\right)$

$= \dfrac{12\pi k\lambda}{R^3} \cdot \dfrac{r^3}{3} \quad \Rightarrow E = \dfrac{2\lambda k r^2}{R^3}$

$r > R$

$E \cdot 2\pi r = \dfrac{12\pi k\lambda}{3} \quad \Rightarrow E = \dfrac{2\lambda k}{r}$

Now it is continuous.