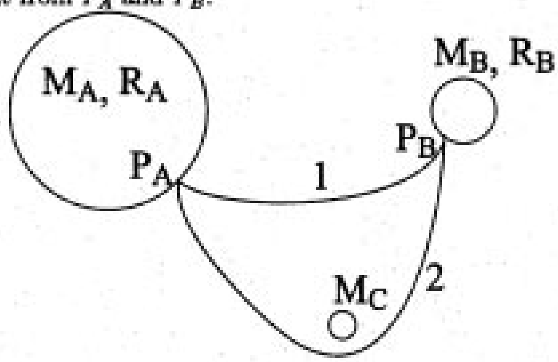
Name Key

Exam #1
Physics 248
February 18, 2004

Each problem is worth 25 points

Problem	Score
1	
2	
3	
4	
Total	

1. Three stars are shown. Light of wavelength λ_A is emitted from point P_A on the surface of star A along the path labelled "1", and arrives at the surface of star B at P_B. Star C is equidistant from P_A and P_B.



(a) Find the wavelength of the light at P_B.

15 pts

$$\Delta\nu/\nu = -\Delta\phi/c^2$$

$$\Delta\phi = \phi_B - \phi_A = -\frac{GM_B}{R_B} - \left(-\frac{GM_A}{R_A}\right) = G\left(\frac{M_A}{R_A} - \frac{M_B}{R_B}\right)$$

(Star C doesn't contribute, since ϕ_C is the same at pts A & B)

$$\Delta\nu = \nu(-\Delta\phi/c^2) \Rightarrow \Delta\lambda = \lambda(\Delta\phi/c^2)$$

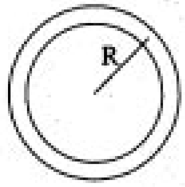
$$\lambda_B = \lambda_A + \frac{G}{c^2}\left(\frac{M_A}{R_A} - \frac{M_B}{R_B}\right)$$

(b) Light of the same wavelength at P_A reaches P_B via path 2. How does it's wavelength compare with the light arriving via path 1?

10 pts

$\Delta\lambda$ only depends on $\Delta\phi$, which is independent of path. So the wavelength for path 2 is the same as path 1.

2. A hollow circular tube is bent into a circle of radius R. If the sound waves have velocity v, what are the three lowest possible standing wave frequencies for sound confined to the interior of the tube?



$$n\lambda = 2\pi R$$

$$v = \lambda \nu \Rightarrow \nu = \frac{v}{2\pi R} \cdot n$$

$$\nu = \frac{v}{2\pi R}, \quad \frac{2v}{2\pi R}, \quad \frac{3v}{2\pi R}$$

3. A particle of mass m is confined by the potential energy function $V(x) = Ax^4/4$.

10 (a) What is the total energy E as a function of the momentum p and the variables given above?

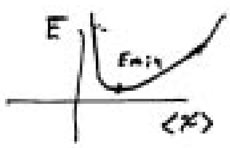
$$E = \frac{p^2}{2m} + \frac{A}{4}x^4$$

5 5

15 (b) Using the uncertainty principle, find the approximate minimum energy for the particle in this potential.

$$5 \quad \Delta p \Delta x \sim \hbar \Rightarrow \langle p^2 \rangle \sim \frac{\hbar^2}{x^2}$$

$$\therefore \; 5 \quad E \sim \frac{\hbar^2}{2mx^2} + \frac{A}{4}x^4$$



$$5 \quad \frac{dE}{dx} \sim \frac{-\hbar^2}{mx^3} + Ax^3 = 0$$

$$\Longleftrightarrow A x^6 = \frac{\hbar^2}{m}$$

$$x = \left(\frac{\hbar^2}{mA}\right)^{1/6}$$

$$\therefore \quad E_{min} \cong \frac{\hbar^2}{2m} \left(\frac{mA}{\hbar^2}\right)^{1/3} + \frac{A}{4}\left(\frac{\hbar^2}{mA}\right)^{2/3}$$

$$\cong \frac{\hbar^{4/3}}{2m^{2/3}} A^{1/3} + \frac{1}{4} A^{1/3} \frac{\hbar^{4/3}}{m^{2/3}}$$

$[A] = \frac{J}{m^4}$

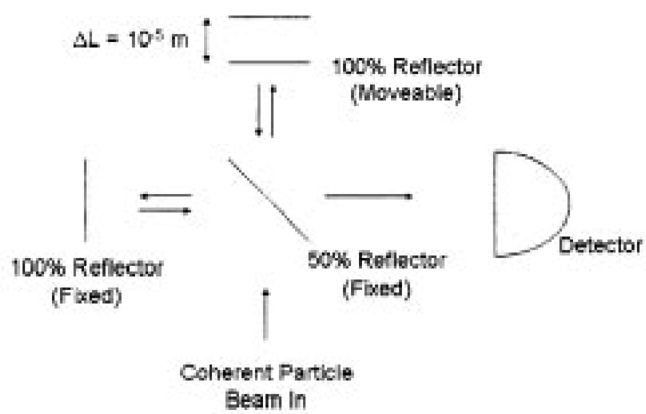
$[\hbar] = J \cdot s$

$[m] = kg$

$$\boxed{E_m \cong \frac{3}{4}\frac{\hbar^{4/3} A^{1/3}}{m^{2/3}}}$$

$$\therefore [E] \to \frac{J^{4/3} s^{4/3} J^{1/3}}{kg^{2/3} m^{4/3}} = J J^{2/3} \frac{s^{4/3}}{kg^{2/3} m^{4/3}} \to \frac{1}{J^{2/3}} = J \checkmark$$

4. The diagram below shows a type of interferometer known as a Michelson Interferometer. Most likely, you have never seen this before, but you know how to analyze it! Waves interfere at the detector. Changing the position of the moveable reflector by 10^{-5} m causes the detector to change from minimum signal to maximum signal. Think about why. Then answer the following three questions. *Use your answer from the first question to answer the second and third questions, even if you have to guess at the first.*



(a) What is the wavelength of the particles?

15

$$2\times10^{-5}\,m = \frac{\lambda}{2}$$

$$\therefore \boxed{\lambda = 4\times10^{-5}\,m}$$

(b) What energy does this correspond to if the particles are photons?

5

$$E = \frac{hc}{\lambda} = \frac{1240\,eV\cdot nm}{4\times10^{-5}\times10^{9}\,nm} = \frac{1240\,eV}{4\times10^{4}} = 310\times10^{-4}\,eV$$
$$= 0.031\,eV$$
$$= 31\,meV$$

(c) What energy does this correspond to if the particles are electrons?

5

$$E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{(hc)^2}{2mc^2\lambda^2} = \frac{(1240\,eV\cdot nm)^2}{1\times10^{6}\,eV\cdot(4\times10^{4}\,nm)^2}$$
$$= \frac{(0.031\,eV)^2}{10^{6}\,eV}$$
$$\approx \frac{9\times10^{-4}}{10^{6}}\,eV = 9\times10^{-10}\,eV$$