Name Solutions

Exam #1

Physics 248

February 15, 2006

Each problem is worth 25 points

Problem	Score
1	
2	
3	
4	
Total	

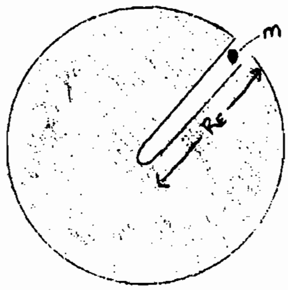
curve $= \frac{15}{26}$ raw $+ 44$

Useful math: $\sin 2\theta = 2\cos\theta\sin\theta$ | $\cos 2\theta = \cos^2\theta - \sin^2\theta$

1 mi=1.609 km | $\sin\theta \approx \theta$ | $\cos\theta \approx 1 - \theta^2/2$

1. Problem 1

Part of the Chamberlin Hall renovation project is to dig a hole from the surface of the earth to its center as shown in the figure. Ignore the earth's rotation and air resistance, and model the earth as a uniform sphere. Express your answers in terms of the acceleration of gravity at the earth's surface g, and the radius of the earth R_E.



(a) How much work is required to lift an object of mass m from the center of the earth to the earth's surface?

$F = \frac{GmM}{r^2}$ where $M = M_E \left(\frac{r}{R}\right)^3 \Rightarrow F = \frac{GmM_E r}{R_E^3} = \frac{mg\,r}{R_E}$

$W = \int_0^{R_E} F\,dr = \frac{mg}{R_E}\frac{R_E^2}{2} = \boxed{\frac{mg\,R_E}{2}}$

(b) If the object is dropped from rest at the surface of the earth, what is its speed when it reaches the center of the earth?

$W = \Delta K = \frac{1}{2}mv^2 \Rightarrow \frac{1}{2}mv^2 = \frac{mg\,R_E}{2}$

$\Rightarrow \boxed{v = \sqrt{g\,R_E}}$

(c) What is the minimum speed required for an object projected from the center of the earth to leave the earth and never return?

Work done in lifting object from center to infinity

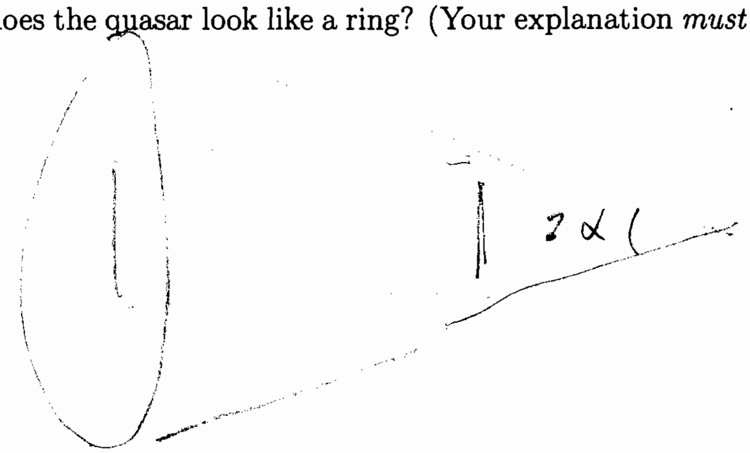
$= \frac{mg\,R_E}{2} + mg\,R_E = \frac{3mg\,R_E}{2}$

$\Rightarrow$ Escape velocity $= \frac{1}{2}mV_e^2 = \frac{3mg\,R_E}{2}$

$V_e = \sqrt{3g\,R_E}$

2. Along a line of sight, we see a galaxy in front of a quasar, as shown. The galaxy has a mass of 10^{11} solar masses, and is 10^{-2} MLy across. We see the quasar as a ring around the galaxy. Note: $M(\text{sun}) = 2 \times 10^{30}$ kg, 1 MLy$= 9 \times 10^{21}$ m.

(a) Why does the quasar look like a ring? (Your explanation *must* include a legible drawing.)



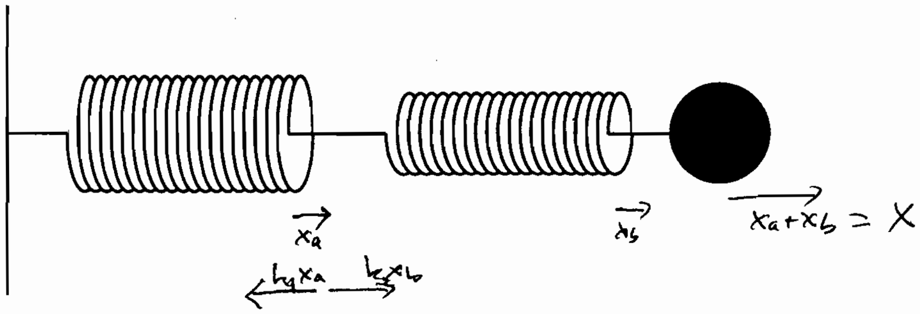
(b) Calculate the angle subtended by the ring in the sky, according to general relativity.

$$2\alpha = 2\left(\frac{4GM}{Rc^2}\right) = 2\left(\frac{4 \times 6.67 \times 10^{-11} \times 2 \times 10^{41}}{4.5 \times 10^{19} \quad (3 \times 10^{8})^2}\right) \text{rad}$$

$$= 2 \times 1.3 \times 10^{-5} \text{ rad}$$

$$= 2.6 \times 10^{-5} \text{ rad}$$

3. A pair of massless springs (spring constants k_1, k_2) have a mass M attached to its end.



(a) Calculate the effective spring constant of the two-spring system. Hint: at the connection between the two springs, there must be a zero net force.

Alt. soln

$k_1 x_a = k_2 x_b$

$X = x_a + x_b$

$\therefore x_b = \frac{k_1}{k_1+k_2} X$

$F = -k_{eff} X = -k_2 x_b = -\frac{k_2 k_1}{k_1+k_2} X$

$$k_1 x_a = k_2 x_b \qquad x_a + x_b = X = \frac{k_2 x_b}{k_1} + x_b = x_b\left(1 + k_2/k_1\right)$$

$$U = \frac{1}{2}\left(k_1 x_a^2 + k_2 x_b^2\right) = \frac{1}{2}\left(k_1 \frac{k_2^2}{k_1^2} x_b^2 + k_2 x_b^2\right) = \frac{1}{2}\left(\frac{k_2^2}{k_1} + k_2\right) x_b^2$$

$$= \frac{1}{2}\left(\frac{k_2^2}{k_1} + k_2\right)\frac{X^2}{\left(1 + k_2/k_1\right)^2} = \frac{\frac{1}{2}\frac{k_2}{k_1}\cancel{(k_2+k_1)}}{\frac{1}{k_1^2}(k_1+k_2)^{\cancel{2}}} X^2$$

$$= \frac{1}{2}\frac{k_1 k_2}{k_1+k_2} X^2 \Rightarrow \frac{1}{K_{eff}} = \frac{1}{k_1} + \frac{1}{k_2}$$

(b) Calculate the oscillation frequency ω_0 of the system.

$$\omega_0 = \sqrt{\frac{K_{eff}}{m}} = \sqrt{\frac{1}{m\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}}$$

(c) The springs are stretched and the ball released. Due to friction, the ball feels a weak damping force $-m\gamma v$, where $\gamma \ll \omega_0$. How many oscillations will the spring make before the oscillations reduce to a value $1/e$ of their original amplitude?

$$\ddot{x} = -\omega_0^2 x - \gamma \dot{x} \qquad x = Ae^{i\alpha t}$$

$$-\alpha^2 = -\omega_0^2 - i\alpha\gamma$$

$$\alpha^2 \approx \omega_0^2 + i\omega_0\gamma$$

$$\alpha = \omega_0\sqrt{1 + \frac{i\omega_0\gamma}{\omega_0^2}} \approx \omega_0\left(1 + \frac{i\omega_0\gamma}{2\omega_0^2}\right) = \omega_0 + i\gamma/2$$

$$\therefore x = e^{i\omega_0 t - \frac{\gamma}{2}t}$$

$$\frac{1}{e} @ \frac{\gamma t}{2} = 1 \Rightarrow t = \frac{2}{\gamma} \qquad \frac{\omega_0 t}{2\pi} = N = \frac{\omega_0}{\gamma\pi}$$

4. A sound wave of intensity I_0 strikes a thin wall. A fraction R of the sound wave intensity is reflected from the wall, while the remainder is transmitted to the next room.

(a) What is the amplitude of the reflected sound wave, as compared to the original?

$$A_0 = \sqrt{I_0}$$

$$A_r = \sqrt{R I_0}$$

(b) What is the amplitude of the transmitted wave, as compared to the original?

$$I_t = (1-R) I_0$$

$$A_t = \sqrt{(1-R) I_0}$$